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F. W. DYSON, Esq., M.A., LL.D., F.R.S., PRESIDENT, in the Chair.

Rev. William Phillips, M.A., Presbyterian Minister, Normanhurst, Clement Avenue, Llandudno, North Wales ; and
John Miles Rogers Todd, Barrister-at-Law, 4 Temple Gardens, Temple, E.C., and The Lodge, Hextable, Kent,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Harry Hinsley Burgham, 34 Aldersgate Street, E.C. (proposed by Frederick W. Bateman) ;

Rev. William Wyatt Holdgate, M.A., Head Master of Sutton Valence School, near Maidstone, Kent (proposed by Albert Taylor) ; and

Harry Russell Maynard, B.A., Kingswood, Antrim Street, Haverstock Hill, N.W. (proposed by Walter Goodacre).

Ninety-nine presents were announced as having been received since the last meeting, including, amongst others :—

Papers and letters connected with the early history of the Society, presented by W. de Morgan, Esq.

Photograph of Spectrum of Arcturus (enlarged transparency),

presented by the Mount Wilson Observatory; photograph of the Kater tablet, Shetland Isles, presented by Major MacMahon; Astrogaphic Chart: 20 charts presented by the Tacubaya Observatory.

A Determination of the Frequency-law of Stellar Motions.

(Plate 2.) By A. S. Eddington, M.A., M.Sc.

(Communicated by the Astronomer Royal.)

It is now well known that the motions of the stars, even after due allowance has been made for the effect of the solar motion, exhibit a strong polarity. There is a tendency to move parallel to a certain line, which is situated in the galactic plane; and both directions, towards either end of this line, are favoured directions of motion. This line cuts the celestial sphere near the point R.A. 94° , Dec. $+12^\circ$, and the opposite point, called the vertex and antivertex; and the stream of stars moving towards the former point is called Stream I., and that towards the opposite point Stream II. The phenomenon is sometimes presented slightly differently by considering the motions of the streams relative to the Sun; from that point of view their directions are inclined at an angle of about 100° ; but, when the solar motion is removed and the stream motions are referred to the centre of mass of the stars concerned, the directions must necessarily be opposite to one another.

In investigating quantitatively the amount and direction of this polarity, two principal forms of representation have been used. The two-drift theory expresses it as if it were due to two systems of stars moving in opposite directions along the line, the internal motions in each system being haphazard. The ellipsoidal hypothesis does not make any division into two systems, but expresses that the average motions of stars in different directions in space are related as the radii of a prolate spheroid, the major axis of the spheroid being in the direction of the polarity aforementioned. I have elsewhere* compared these two representations, showing that, although the expressions differ in mathematical form, it is possible for them to embody almost indistinguishable distributions of velocity.

In the present paper I have tried to determine what is the actual distribution or frequency-law of the linear velocities without assuming that it must be of either of these forms. This frequency-law is to be deduced from the observed distribution of the angular motions, not only as regards direction, but as regards both amount and direction. I had hoped that it might be possible to avoid making any assumption whatever as to the form of the frequency-law; but I was ultimately obliged to limit it to a form which, however, was sufficiently general to cover as particular cases the

* *Brit. Assoc. Report*, 1911.

ellipsoidal, two-drift, and Halm's three-drift theories. All these theories assume that the component motions at right angles to the line of polarity are distributed according to the law of accidental errors. I have used this property so far as concerns those stars whose whole motion (referred to the Sun) is at right angles to the line of polarity; in all other directions no limitation is imposed.

The result arrived at in this paper is opposed to the ellipsoidal and in favour of the two-drift (not excluding the three-drift) representation. The whole investigation, however, is somewhat tentative, and it is difficult to say how much reliance must be placed on any particular deduction. I feel sure that no minor difference in the mathematical treatment could make these particular data yield a result favourable to the ellipsoidal theory; for, before the solution now published was reached, three other nearly completed attempts were made, all of which indicated a two-drift distribution in the same way. But it may legitimately be urged that the data have been insufficient to give a great accuracy of detail in the resulting frequency-law, and I should rather hesitate to assert that the particular detail which distinguishes the rival mathematical hypotheses is entirely trustworthy. The conclusion on this particular point is perhaps disappointing; these results definitely favour the view that the stars are grouped in two (or three) distinct systems, but can hardly be considered to settle the question conclusively.

The principal results are given in a graphical form in the diagram (Plate 2). This gives the actual distribution of the linear motions determined by this method. The distribution of the stars as regards distance, which is incidentally determined (p. 386), is also of interest; but it should be received with even greater caution.

Mathematical Theory.

We assume that the law of distribution of the linear velocities of the stars does not depend on the distance of the stars considered from the solar system.

We have to consider the relation of three frequency-laws—

- (1) The law of distribution of the linear motions of the stars.
- (2) The law of distribution of the proper or angular motions of the stars.
- (3) The distribution, as regards distance along the line of sight, of the stars of the catalogue.

These three laws are connected by an integral equation, from which, if any two of them are given, the third can be found. The catalogue of proper motions gives us (2); but to find (1) we must also know (3), or *vice versa*. My method was this:* as the

* I had expected to make use of a quite extraneous determination of law (3) by Professor Kapteyn, but that was found to be irreconcilable with my data and could not be used. It was therefore necessary to include the determination of (3) in my investigation.

linear motion relative to the Sun has the same direction as the proper motion, we can consider separately the proper motions in any direction. I chose the direction perpendicular to the line of polarity, in which the ellipsoidal and two-drift theories agreed that the law (1) was the law of errors (there is also a component of solar motion in that direction which must be taken into account). Knowing then (1) and (2) for this direction, I determined (3). But (3) applies to all the other directions, and therefore, knowing (2) and (3) for the other directions, I determined (1) for them. This completed the solution.

Confine attention to a limited area of the sky, which can be treated as plane, and to stars moving in directions within any given limits of position angle.

Let $g(u)\frac{du}{u}$ be the number of stars having linear motions between u and $u + du$.

Let $h(a)\frac{da}{a}$ be the number having proper motions between a and $a + da$.

Let $f(r)\frac{dr}{r}$ be the proportion of stars (of the catalogue) comprised within the limits of distance from the Sun r and $r + dr$, so that

$$\int_0^\infty f(r)\frac{dr}{r} = 1.$$

Then, expressing that $h(a)\frac{da}{a}$ is made up of stars at all possible distances r , and with linear motions u such that u is between ra and $r(a + da)$, it is easily seen that

$$h(a)\frac{da}{a} = \int_0^\infty f(r)\frac{dr}{r} \cdot \frac{g(ra)}{ra} r da.$$

Hence
$$h(a) = \int_0^\infty f(r)g(ra)\frac{dr}{r} \quad . \quad . \quad . \quad (1)$$

For the solution of this integral equation, the method given by Prof. Schwarzschild (*Ast. Nach.*, No. 4422) for a somewhat similar problem can be used. Write

$$r = e^\lambda, \quad a = e^\mu, \quad \text{and} \quad u = e^\gamma,$$

$$f(e^\lambda) = f_1(\lambda), \quad g(e^\gamma) = g_1(\gamma), \quad h(e^\mu) = h_1(\mu).$$

Equation (1) becomes

$$h_1(\mu) = \int_{-\infty}^\infty f_1(\lambda)g_1(\lambda + \mu)d\lambda \quad . \quad . \quad . \quad (2)$$

Let us form the Fourier integrals

$$\left. \begin{aligned} F(q) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} f_1(\lambda) e^{-iq\lambda} d\lambda \\ G(q) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} g_1(\lambda) e^{-iq\lambda} d\lambda \\ H(q) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} h_1(\lambda) e^{-iq\lambda} d\lambda \end{aligned} \right\} \quad . \quad . \quad . \quad (3)$$

Then we have the well-known reciprocal relation

$$\left. \begin{aligned} f_1(\lambda) &= \int_{-\infty}^{\infty} F(q) e^{iq\lambda} dq \\ g_1(\lambda) &= \int_{-\infty}^{\infty} G(q) e^{iq\lambda} dq \\ h_1(\lambda) &= \int_{-\infty}^{\infty} H(q) e^{iq\lambda} dq \end{aligned} \right\} \quad . \quad . \quad . \quad (4)$$

From (2) we have

$$\begin{aligned} \int_{-\infty}^{\infty} h_1(\mu) e^{-iq\mu} d\mu &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(\lambda) g_1(\lambda + \mu) e^{-iq\mu} d\lambda d\mu \\ &= \int_{-\infty}^{\infty} f_1(\lambda) e^{iq\lambda} d\lambda \int_{-\infty}^{\infty} g_1(\lambda + \mu) e^{-iq(\lambda + \mu)} d\mu \\ &= 2\pi F(-q) \cdot 2\pi G(q). \end{aligned}$$

$$\text{Hence} \quad H(q) = 2\pi F(-q) G(q) \quad . \quad . \quad . \quad . \quad (5)$$

This is the solution of the problem; for if the functions f_1 and h_1 are given, we can form the functions F and H , calculate G from (5), and deduce g_1 from G by means of (4).

The process may be summarised by saying that we form periodograms of the curves for f_1 and h_1 , divide the second by the first, and form the periodogram of the result.*

There are several points of difficulty in the practical application of this solution.

We may first notice a result by Prof. Seeliger, which was kindly pointed out to me by Prof. Schwarzschild. If the density of distribution of the stars at a distance r from the solar system varies as $r^{-\kappa}$, so that

$$f(r) = Cr^{3-\kappa},$$

where C is constant, then by (1)

$$h(a) = C \int_0^{\infty} r^{2-\kappa} g(ra) dr;$$

or, writing $u = ra$,

$$h(a) = Ca^{\kappa-3} \int_0^{\infty} u^{2-\kappa} g(u) du.$$

* The periodogram must be understood to be a complex quantity expressing the phase as well as the amplitude.

But the integral on the right-hand side is independent of a , and hence

$$h(a) \propto a^{\kappa-3},$$

whatever may be the frequency-law of the linear motions. In such a case the solution for g clearly must be indeterminate.

It would, of course, be quite impossible for the distribution of the stars of the catalogue to obey the density law $r^{-\kappa}$ for all values of r , since that would involve an infinite number of stars close to the Sun if $\kappa > 3$, or an infinite number of very distant stars if $\kappa < 3$. But if such a law held for a large part of the range, we could expect only a weak solution of the equations. In the first place, therefore, it is desirable to see how nearly $h(a)$ approximates to the form $a^{\kappa-3}$.

In Table I. the material of this investigation is divided to show how many of the total proper motions are between $0''$ and $1''$, $1''$ and $2''$, etc., per century. In our notation this is $h(a)/a$. Inspection shows that no law of the form proposed fits the data for any considerable range; perhaps as good an approximation as possible is given by taking $\kappa = 2.14$, so that $h(a)/a \propto a^{-1.86}$. (This is determined by making the formula agree with observation near $a = 6''$ and $a = 22''$.) The calculated numbers are given in the third column. The failure of the formula for $a < 15''$ and $a > 40''$ is so considerable that we conclude there is no approach to the indeterminate case, and we may expect that the data will yield a fairly good determination of the function g .

TABLE I
Magnitudes of Proper Motions.

Limits of Centennial P.M.	No. of Stars.	Trial Formula.	Limits of Centennial P.M.	No. of Stars.	Trial Formula.
0- 0'9	33	...	" " " 13-13'9	26	18
1- 1'9	95	...	14-14'9	19	16
2- 2'9	112	420	15-15'9	16	14
3- 3'9	112	223	16-16'9	15	12
4- 4'9	92	139	17-17'9	10	11
5- 5'9	84	95	18-18'9	8	10
6- 6'9	83	69	19-19'9	11	9
7- 7'9	70	53			
8- 8'9	56	42	20-25	38	34
9- 9'9	45	34	25-30	16	24
10-10'9	32	29	30-35	27	17
11-11'9	39	24	35-40	17	14
12-12'9	23	20	> 40	54	108

Considerable difficulty is caused by the liability of the integrals to diverge. According to the equations (3), (4), and (5), the value of $g_1(\gamma)$, where $\gamma = \log u$, is

$$g_1(\gamma) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} e^{iq\gamma} \frac{dq}{F(-q)} \int_{-\infty}^{\infty} h_1(\mu) e^{-iq\mu} d\mu. \quad (6),$$

where the distribution of the stars as regards distance is assumed to be known, and therefore $F(-q)$ is known.

Now the observed $h_1(\mu)$, as given directly by the proper motions of a finite number of stars, is a discontinuous function. Strictly speaking, if there are n stars, and we divide the range into a very large number of elements $\delta\mu$, $h_1(\mu)\delta\mu$ takes the value unity for n isolated elements, corresponding to the n proper motions, and is zero everywhere else. But if $h_1(\mu)$ is discontinuous in this manner, the Fourier Integral $\int_{-\infty}^{\infty} h_1(\mu) e^{-iq\mu} d\mu$ does not tend to zero for large values of q , but oscillates rapidly, and when divided by $F(-q)$ becomes completely divergent; the solution thus breaks down. This is only to be expected; because there can be no exact solution of the problem in this case, and our formulæ are unfortunately not adapted to give an approximate solution when no exact one is possible.

We should naturally prefer to postpone any smoothing required until the values of $g_1(\gamma)$ had been arrived at; but we are compelled to start by smoothing $h_1(\mu)$, and making it into a continuous function. If this is done, no difficulty will arise in calculating the integral $H(q)$; but even then it may, and often does, happen that the division by $F(-q)$ causes divergence and makes the second integration impossible. To show under what conditions this happens, let us evaluate $g_1(\gamma)$ in a comparatively simple case.

Suppose

$$\left. \begin{aligned} f_1(\lambda) &= \sqrt{\frac{\xi}{\pi}} e^{-\xi(\lambda - \sigma)^2} \\ h_1(\mu) &= N \frac{\kappa}{\sqrt{\pi}} e^{-\kappa^2(\mu - \mu_0)^2} \end{aligned} \right\} \quad (7)$$

The constants σ and μ_0 could be made zero by suitably choosing the units of distance and proper motion, but it is convenient to retain them. N is the number of stars considered.

The expressions for h_1 and f_1 actually used later in this paper are more complicated; but the error-curves given by (7) are sufficiently good approximations to give a valuable insight into the conditions of solution.

We have from (3)

$$2\pi F(q) = \int_{-\infty}^{\infty} \sqrt{\frac{\xi}{\pi}} e^{-\xi(\lambda - \sigma)^2 - iq\lambda} d\lambda,$$

which reduces to *

$$2\pi F(q) = \exp. \left\{ -\frac{q^2}{4\xi} - iq\sigma \right\} . \quad (8)$$

Similarly

$$2\pi H(q) = N \exp. \left\{ -\frac{q^2}{4\kappa^2} - iq\mu_0 \right\} . \quad (9)$$

Therefore

$$2\pi G(q) = \frac{H(q)}{F(-q)} = N \exp. \left\{ -q^2 \left(\frac{1}{4\kappa^2} - \frac{1}{4\xi} \right) - iq(\mu_0 + \sigma) \right\} .$$

And hence by (4)

$$g_1(\gamma) = \frac{N}{2\pi} \int_{-\infty}^{\infty} \exp. \left\{ -q^2 \left(\frac{1}{4\kappa^2} - \frac{1}{4\xi} \right) - iq(\mu_0 + \sigma - \gamma) \right\} dq.$$

This reduces to

$$g_1(\gamma) = \frac{N}{\sqrt{\pi \left(\frac{1}{\kappa^2} - \frac{1}{\xi} \right)}} \exp. -\frac{(\gamma - \mu_0 - \sigma)^2}{\frac{1}{\kappa^2} - \frac{1}{\xi}} . \quad (10)$$

provided

$$\frac{1}{\kappa^2} > \frac{1}{\xi} . \quad (11)$$

If the condition $\frac{1}{\kappa^2} > \frac{1}{\xi}$ is not fulfilled, the solution is divergent.

This is a useful guide, by which we may avoid divergent solutions in the general case. Equation (7) gives to $h_1(\mu)$ the form of an error-curve, and the smaller the value of $\frac{1}{\kappa^2}$ the more sharply peaked will this curve be. In order that a solution may be possible, the peak must not be too sharp, as is shown by (11). When $h_1(\mu)$ is not assumed to be an error-curve, we must still take care that there are no peaks or roughnesses which are sharper than a certain limit; if the data of observation appear to indicate too abrupt a peak, it shows that either the assumed law for $f_1(\lambda)$ is wrong, or the peak is a mere accident and must be smoothed down to come within the necessary limit.

In finding $g_1(\gamma)$ from equation (6) solutions are additive. Apparently we may arbitrarily divide the given distribution of proper motions, expressing it as the sum of two or more distributions, determine the distribution of linear motions corresponding to each, and add together the results. But the necessity of performing the division so that all the integrals are convergent imposes a very strict limitation in practice.

* The general formula for reducing these expressions is

$$\int_{-\infty}^{\infty} e^{-b_1 x - b_2 x^2} dx = \sqrt{\frac{\pi}{b_2}} \exp. \frac{b_1^2}{4b_2},$$

subject to obvious conditions of convergency.

When $f_1(\lambda)$ is an error-function as in (7), a complete algebraic solution can be found when $h_1(\mu)$ is of the form

$$(a_0 + a_1(\mu - \mu_0) + a_2(\mu - \mu_0)^2 + \dots)e^{-\kappa^2(\mu - \mu_0)^2}.$$

It happens that the data of observation can usually be expressed very closely in the form

$$h_1(\mu) = (a_0 + a_4(\mu - \mu_0)^4)e^{-\kappa^2(\mu - \mu_0)^2},$$

which is of the above type; but the solution turns out to be useless, as the value found for $g(u)$ makes wide oscillations. The fact is that we have no means of imposing the condition that $g(u)$ is to be always positive and that it is to be a smooth curve, and we are always liable to obtain solutions which disregard these conditions. Thus, in place of finding 10, 12, and 14 stars with velocities of 1, 2, and 3 km./sec. respectively, our solution may give 30, -28, and 34 stars with those velocities. It is too scrupulously exact, and, in place of disregarding trivial roughnesses in the observed data, makes the most astounding oscillations in the attempt to represent them exactly.

As the result of a great deal of experiment, I find that the safest course is to express $h_1(\mu)$ as the sum of a number of error-curves: generally three are found necessary. If care is taken that for each of them $\frac{1}{\kappa^2}$ is above a certain limit,* the solution for $g_1(\gamma)$ will be everywhere positive and free from oscillations.

Application to the observed Data.

The material consists of the stars of Professor Boss's Preliminary General Catalogue (excluding those of the Orion type) contained within the two opposite areas

R.A. $2^h 36^m$ to $2^h 24^m$, Dec. -36° to $+36^\circ$
and R.A. $9^h 36^m$ to $14^h 24^m$, " " "

This corresponds to Regions VIII., XII., XIII., and XVII. of my previous investigation.

The whole number of stars is 1129.

The centres of the areas are on the equator at 0^h and 12^h , so that we are really investigating the distribution of linear motions in planes parallel to that which cuts the sphere in the meridians of 6^h and 18^h , neglecting altogether the components perpendicular to this plane. This area of the sky might appear to be unduly large to treat as plane; but, owing to the particular choice of plane, I think no error will arise from this cause. Both the solar apex and the vertex are approximately 90° from the centre of the region, and therefore, in projecting on the tangent plane, the lines joining all the different points of the region to either

* This limit is gradually found by practice. It is somewhat greater than $\frac{1}{\xi}$, because, when $\frac{1}{\kappa^2}$ is only just greater than the critical value, the solution, though convergent, is unsuitable.

apex or vertex project into nearly parallel lines. This would not be true in any other part of the sky; the directions towards the vertex or apex, or both, would converge, and in so large an area the inclination would be very considerable. It follows that near this particular point it is permissible to use a much larger area of the sky than could be admitted elsewhere. Whilst this consideration necessarily determined the choice, the region was especially suitable for another reason. Being near the galactic poles, the proper motions are on the average large, and therefore proportionately more accurately determined than in lower galactic latitudes. Further, in this part of the sky the full effect of the star-streaming is seen without foreshortening.

Having calculated the magnitude and direction of each proper motion, I grouped them according to direction in 30° sectors, *i.e.* stars moving in the directions $335^\circ-5^\circ$, $5^\circ-35^\circ$, $35^\circ-65^\circ$, etc. The direction θ is measured so that $\theta=0$ is towards the point R.A. 6^h Dec. 0° , and $\theta=90^\circ$ is towards the South Pole. Table II. shows, for each sector, the number of stars for which the decimal logarithm of the proper motion (unit= $0''.001$ per annum) lies between the limits given in the first column. This is practically the function $h_1(\mu)$, only that the decimal logarithm μ' is used instead of the natural logarithm μ .

TABLE II.
Number of Proper Motions within given Limits.

μ'	Sectors.											
($=\log_{10}$ P.M.)	5° to 35°	35° to 65°	65° to 95°	95° to 125°	125° to 155°	155° to 185°	185° to 215°	215° to 245°	245° to 275°	275° to 305°	305° to 335°	335° to 5°
< 1.0	2	5	3	4	5	1	2	2	2	4	4	1
1.0-1.1	1	4	4	1	1	1	1	0	1	1	0	2
1.1-1.2	8	4	5	3	4	2	1	1	0	1	0	3
1.2-1.3	8	8	4	1	3	4	1	0	4	2	5	4
1.3-1.4	10	6	5	5	6	6	1	0	5	1	3	9
1.4-1.5	15	13	6	5	15	4	0	1	4	5	4	9
1.5-1.6	7	14	13	11	5	6	4	1	2	4	3	15
1.6-1.7	28	12	8	12	5	4	3	0	1	4	8	13
1.7-1.8	22	22	8	7	15	9	0	2	0	2	5	18
1.8-1.9	21	28	10	8	12	4	0	0	3	0	2	33
1.9-2.0	32	14	7	3	13	3	2	2	1	0	1	25
2.0-2.1	20	12	7	8	3	5	1	0	0	1	3	25
2.1-2.2	19	9	1	4	6	4	1	1	1	1	3	15
2.2-2.3	11	4	4	4	3	1	0	0	0	1	4	11
2.3-2.4	13	4	3	5	6	0	0	0	2	1	2	8
2.4-2.5	11	0	1	4	0	0	0	0	0	2	0	8
2.5-2.6	5	7	3	2	6	0	0	1	0	0	1	7
2.6-2.7	10	0	4	2	0	0	0	0	0	0	0	7
> 2.7	1	6	2	2	2	2	1	0	1	0	4	7
Total	244	172	98	91	110	56	18	11	27	30	52	220

If we smooth the columns and plot the results, we obtain curves which at once suggest the error-curve. The curves are almost symmetrical about the maximum ordinate. This could hardly have been anticipated *a priori*, and is a very fortunate circumstance from the analytical standpoint. On attempting to fit an error-curve to the observations, we find that, although it is a very fair first approximation, there is a quite definite difference. If the whole error-curve is divided by ordinates into six parts of equal area, the observed curve falls below it in the 2nd and 5th segments, and rises above it in the two extreme and two middle segments. This was found to be true in all cases.

The most natural modification is to compare the observed curve with $y = (a_0 + a_4 x^4) e^{-\kappa^2 x^2}$ instead of the simple error-curve $y = e^{-\kappa^2 x^2}$; but, as already explained, the solution arrived at in this way turns out to be an oscillating one. It is not possible to represent the observed distribution by the sum or difference of two error-curves, whether coaxial or not. The method I finally adopted was to use three error-curves, of which one principal curve represents the majority of the motions, another provides for the excess of very small proper motions, and a third provides for the excess of very large motions. A very good approximation can be obtained in this way. In the sectors, which contain less than 50 stars, a single error-curve was deemed sufficient.

Representing, then, $h_1(\mu)$ as the sum of 3 terms, thus—

$$h_1(\mu) = \sum_{r=1,2,3} N_r \frac{\kappa_r}{\sqrt{\pi}} e^{-\kappa_r^2 (\mu - \mu_r)^2},$$

or, using the corresponding decimal logarithms μ' and μ_r' ,

$$h_1(\mu) = \sum_{r=1,2,3} .434 N_r \frac{\kappa_r'}{\sqrt{\pi}} e^{-\kappa_r'^2 (\mu' - \mu_r')^2};$$

the coefficients in the latter expression are given in Table III. Here N_1, N_2, N_3 are the number of stars in the three components into which the distribution is divided (so that $N_1 + N_2 + N_3$ is approximately equal to the observed number of stars moving in the sector); μ_1', μ_2', μ_3' mark the positions of the axes of the three component error-curves; and $\kappa_1', \kappa_2', \kappa_3'$ are their moduli. Also $\kappa_r = .434 \kappa_r'$ and $\mu_r = \mu_r' / .434$.

From these $H(q)$ can be determined by (9), viz.

$$2\pi H(q) = \sum_{r=1,2,3} N_r \exp. \left(-\frac{q^2}{4\kappa_r} - iq\mu_r \right)$$

An example of the function $H(q)$ thus calculated is given later (Table IV.); a certain constant multiplier has been ignored. q is given in degrees, but the corresponding value in radians must be used in the formula.

TABLE III.

Coefficients of the Formula used to represent the observed P.M.'s.

Sector.	Coefficient.	$r=1.$	$r=2.$	$r=3.$
$335^{\circ}-5^{\circ}$	N_r	171.5	18.4	29.8
	μ_r'	1.90	1.30	2.575
	$1/\kappa_r'$	.40	.40	.40
$5^{\circ}-35^{\circ}$	N_r	207.0	19.1	19.3
	μ_r'	1.875	1.40	2.525
	$1/\kappa_r'$	.47	.45	.34
$35^{\circ}-65^{\circ}$	N_r	134.6	26.2	10.0
	μ_r'	1.80	1.25	2.65
	$1/\kappa_r'$	.40	.40	.40
$65^{\circ}-95^{\circ}$	N_r	83.0	4.8	9.9
	μ_r'	1.675	0.975	2.55
	$1/\kappa_r'$	.50	.34	.57
$95^{\circ}-125^{\circ}$	N_r	65.2	5.7	16.5
	μ_r'	1.725	0.10	2.40
	$1/\kappa_r'$	.40	.40	.42
$125^{\circ}-155^{\circ}$	N_r	92.5	7.9	12.1
	μ_r'	1.75	1.05	2.425
	$1/\kappa_r'$	.45	.45	.38
$155^{\circ}-185^{\circ}$	N_r	48.0	4.2	6.0
	μ_r'	1.675	1.175	2.20
	$1/\kappa_r'$	.41	.34	.34

The remaining sectors (containing very few stars) were represented by simple error-curves.

Coefficients.

	$185^{\circ}-215^{\circ}$.	$215^{\circ}-245^{\circ}$.	$245^{\circ}-275^{\circ}$.	$275^{\circ}-305^{\circ}$.	$305^{\circ}-335^{\circ}$.
N.	18.1	11.0	27.0	30.1	52.1
μ_1'	1.55	1.625	1.45	1.55	1.70
$1/\kappa'$	.64	.76	.60	.62	.71

As explained in the introductory paragraphs, I assumed that in the direction at right angles to the line joining the vertex and antivertex, the linear motions were distributed according to Maxwell's law (after abstracting the component of solar motion in that direction). The ellipsoidal, two-drift, and three-drift theories are all in agreement with this assumption. The direction of the vertex is near $\theta = 348^{\circ}$, so that the sector $\theta = 65^{\circ} - 95^{\circ}$ and the opposite one are those to which this assumption is applicable. If τ be the component of the solar motion in the direction $\theta = 80^{\circ}$ (expressed in terms of the usual unit $\frac{1}{h}$, which depends on the mean peculiar velocities), we have

$$\frac{\text{No. of stars in the sector } 65^{\circ}-95^{\circ}}{\text{No. of stars in the sector } 245^{\circ}-275^{\circ}} = \frac{f(\tau)}{f(-\tau)}$$

where $f(\tau)$ is the quantity tabulated in my paper "Systematic Motions of the Stars" (*M. N.* lxxvii. p. 37).

We find that the ratio is $\frac{98}{27}$, which leads to $\tau = \cdot 36$. The determination is, of course, a very weak one, and there is reason to think it is a rather small value. Using the constants found from an analysis of the whole catalogue, $\tau = \cdot 65$; but the higher proportion of Type II. stars in this particular region, which have large peculiar motions, would probably alter the unit $\frac{1}{h}$, and thus reduce the appropriate value of τ . Ultimately I started two solutions, one with the value $\tau = \cdot 43$ and the other for $\tau = \cdot 56$. After some trial the former was thought to be most satisfactory, and was completed; the latter was carried far enough to test the effect on the general character of the results.

Assuming then $\tau = \cdot 43$, we have (omitting a constant multiplier)

$$\frac{g(u)}{u}du = ue^{-(u-\cdot 43)^2}du$$

for the sector $65^{\circ}-95^{\circ}$; and therefore putting $u = e^{\gamma}$,

$$g_1(\gamma) = e^{2\gamma}e^{-(e^{\gamma}-\cdot 43)^2}.$$

This was tabulated and the Fourier Integral $G(q)$ calculated (partly by formulæ and the residuum by quadratures).

We have now $H(q)$ and $G(q)$ for this sector, and $F(-q)$ is obtained by division. These are given in Table IV.; in all cases constant multipliers are disregarded.

TABLE IV.
Sector 65° to 95°.

$q.$			$H(q).$			$G(q).$			$F(-q).$		
0°	55'2	expi *	0°	9'72	expi	0°	5'68	expi	0°		
25	49'4	„ -	99'0	9'41	„ +	1'2	5'25	„ -	100'2		
50	36'2	„ -	195'4	8'57	„ +	1'2	4'22	„ -	196'6		
75	23'2	„ -	288'1	7'45	„ -	0'4	3'11	„ -	287'7		
100	14'1	„ -	381'3	6'26	„ -	4'0	2'25	„ -	377'3		
125	8'29	„ -	480'6	5'15	„ -	9'4	1'61	„ -	471'2		
150	4'60	„ -	585'9	4'16	„ -	16'6	1'105	„ -	569'3		
175	2'48	„ -	689'5	3'31	„ -	25'0	'750	„ -	664'5		
200	1'24	„ -	785'0	2'60	„ -	34'9	478	„ -	750		
225	'519	„ -	866'5	2'06	„ -	46'2	'252	„ -	820		
250	'219	„ -	940'8	1'59	„ -	58'5	'138	„ -	882		
275	'094	„ -	1001	1'20	„ -	72'3	'078	„ -	929		
300	'040	„ -	1041	'892	„ -	87'5	'045	„ -	953		

* $\text{expi } \theta \equiv e^{\theta}$.

Since $F(-q)$ expresses the distribution of the stars as regards distance, the same value is applicable to all the sectors. We therefore work out $2\pi G(q) = \frac{H(q)}{F(-q)}$ for all the other sectors and obtain by quadratures (or Fourier Analysis) for each of them

$$g_1(\gamma) = \int_{-\infty}^{\infty} G(q)e^{iq\gamma}dq.$$

The results are given in Table V. This is a table analogous to Table II. showing for each sector the number of stars for which the logarithm of the *linear* motion (in an arbitrary unit) lies between the limits γ and $\gamma + d\gamma$ corresponding to the value of γ given in the first column. The logarithms are to the base e .

TABLE V.

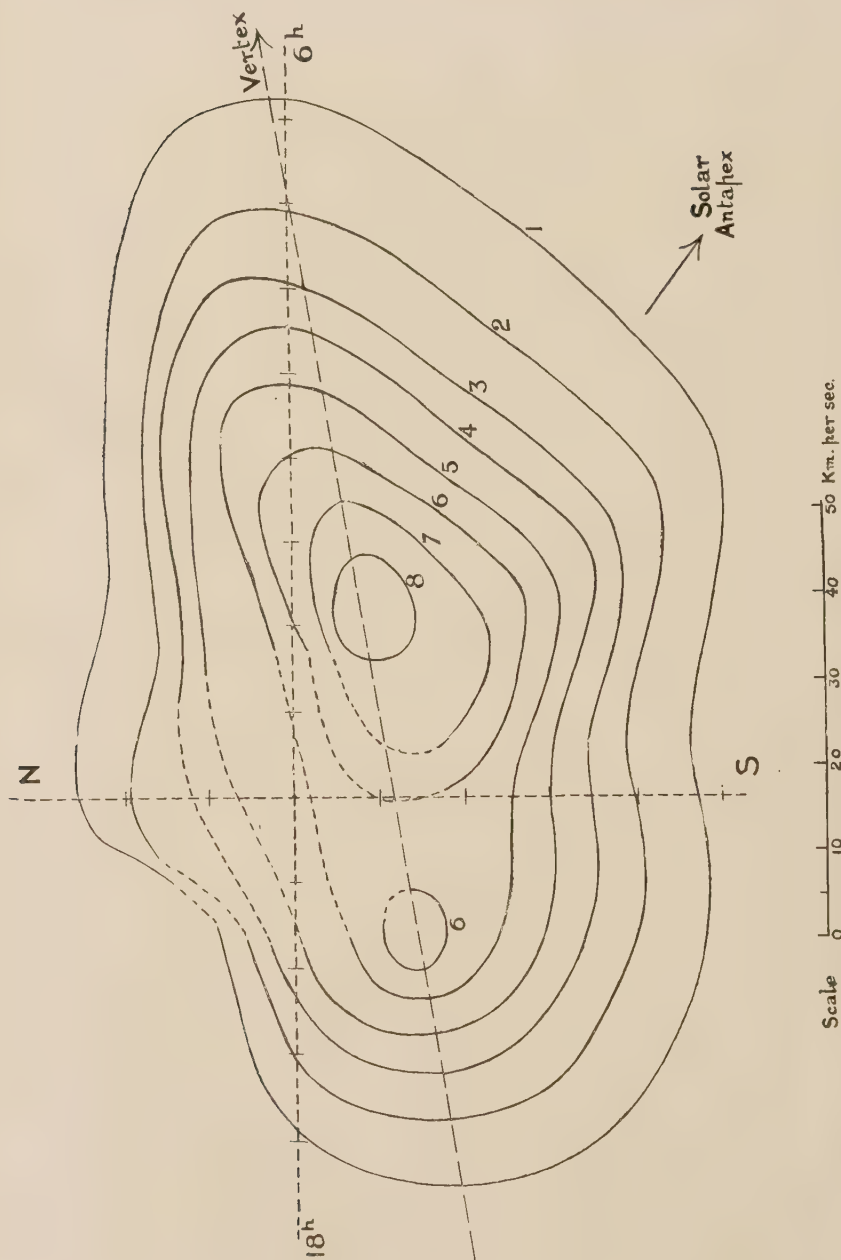
Values of $g_1(\gamma)$ in the twelve sectors.

γ .	5° to 35° .	35° to 65° .	65° to 95° .	95° to 125° .	125° to 155° .	155° to 185° .	185° to 215° .	215° to 245° .	245° to 275° .	275° to 305° .	305° to 335° .	335° to 5° .
-1.2	...	...	6.8	...	...	...	...	2.6	7.3	...	...	...
-1.0	...	...	10.2	...	2.9	2.9	4.6	2.9	9.2	7.7	10.0	...
- .8	5.3	12.8	15.2	-4.9	4.5	8.0	5.6	3.2	10.9	9.7	12.4	3.4
- .6	15.9	16.5	22.4	5.0	11.1	14.4	6.5	3.3	12.0	11.3	14.4	4.5
- .4	33.8	28.6	31.9	19.8	26.1	21.5	7.0	3.3	12.1	12.4	16.0	8.2
- .2	58.9	47.7	43.4	36.7	43.0	30.3	7.2	3.1	11.4	12.7	16.9	19.6
.0	89.1	74.6	54.4	52.7	60.0	38.3	6.9	2.9	9.9	12.1	16.9	45.0
.2	117.9	105.3	60.1	63.9	71.3	42.9	6.2	2.6	7.8	10.8	16.1	84.9
.4	138.3	129.2	54.3	65.1	71.8	40.2	5.1	2.2	5.5	8.8	14.6	131.7
.6	143.0	132.5	36.1	52.5	59.5	28.7	4.0	...	...	6.5	12.6	168.2
.8	128.6	104.3	14.9	28.0	37.6	11.7	...	...	...	...	10.3	175.4
1.0	96.0	47.4	2.9	0.9	13.5	...	...	...	...	...	7.9	140.4
1.2	33.7	-17.2	...	...	-4.3	...	...	...	...	...	...	69.6

The result of omitting constant multipliers at various stages of the work is that the figures in this table are 3.6 times too great, compared with the actual number of stars observed. The multiplier is of no theoretical importance; but in estimating the significance of any discrepancy, it may be useful to remember that 3.6 in any of the numbers entered in Table V. is equivalent to a single star in the observed data.

It now remains to reduce the results to a convenient form for illustrating the law of distribution of the linear motions. The frequency $D(\xi, \eta)$ of a linear motion whose rectangular components are (ξ, η) is defined by the relation:—

Number of stars with linear motion between ξ and $\xi + d\xi$, and η and $\eta + d\eta = D(\xi, \eta)d\xi d\eta$.



DISTRIBUTION OF LINEAR VELOCITIES OF STARS.—A. S. EDDINGTON.

Turning to the definition of g , we see that the frequency is $\frac{g(u)}{u^2}$, or $g_1(\gamma)e^{-2\gamma}$, allowing for the increase in width of the sector as u increases.

To exhibit the frequency of different values of the linear motion we draw the curves of equal frequency and obtain the figure (Plate 2). The curves are drawn through points determined from Table V. on the twelve equidistant radii; all the irregularities have been allowed to remain. But no attempt has been made to determine points within a certain distance of the origin ($\gamma < -\cdot 35$), because, owing to the increase of the factor $e^{-2\gamma}$, the frequency depends on more and more slender material as we approach the origin. Within the circle the equifrequentals have been joined up by the dotted lines in the way which seemed best to complete the diagram.

Explanation of the Diagram.

The diagram refers to the plane through the North and South Poles and the meridians of 6^h and 18^h R.A., and represents the distribution of stellar velocities in that plane, the components perpendicular to it being ignored. Any particular speed and direction is represented in the usual way by a point on the diagram, and the frequency of that motion (*i.e.* the proportionate number of stars having that motion) is indicated by contour-lines (or equifrequentals) drawn on the diagram. Starting from the outside of the figure, the successive curves correspond to the relative frequencies 1, 2, 3, 4, 5, 6, 7, 8. The most favoured values of stellar velocities are those which correspond to the points within the equifrequental line 8. The scale of velocity is given below the diagram, and, in addition, intervals corresponding to 10 km./sec. are set off along the axes of coordinates.

The curves are like the contour-lines of a region with two peaks; the height of the peak on the right would be almost 9.0, that on the left 6.4, and the pass between them 5.4. The meaning of these curves may be made clearer by some examples. Suppose it is required to compare the number of stars moving in each case with a velocity of 25 km./sec. towards (1) North Pole, (2) South Pole, (3) R.A. 6^h on the equator, (4) R.A. 18^h on the equator. Estimating from the curves near the corresponding points, just as we should estimate heights from the contours on a map, we find the relative numbers are

$$1.0, 5.1, 6.3, 2.7.$$

Again, the equifrequental line 3 passes through points corresponding to velocities 60 km./sec. towards R.A. 6^h , and 23 km./sec. towards R.A. 18^h ; these values of the velocity accordingly occur with equal abundance. Similarly, a little measurement shows that a velocity 57 km./sec. towards R.A. 6^h Dec. -45° should occur just as often as a velocity of 15 km./sec. in the opposite direction. Again, for every star moving towards the South Pole with velocity

47 km./sec., there are five moving in the same direction with velocity 26 km./sec.

In all these examples it is understood that components of motion perpendicular to the plane of the diagram are ignored, and by the number of stars having a given motion is meant the number whose motions differ from it (geometrically) by less than a fixed small amount. The motions are taken relatively to the Sun.

It will be noticed that the origin of coordinates bears no obvious relation to the arrangement of the curves. This was to be expected, since the origin is simply the point which represents the motion of the Sun, and has no cosmical importance. If we leave out the origin, the diagram becomes perfectly general and represents the true cosmical arrangement of the motions of stars. The origin, however, has enabled us to determine the scale of the figure, since the distance of the origin from the axis of the figure is a component of the solar motion whose value in kilometres per second is known approximately.

The very noticeable elongation of the distribution represents the now well-known polarity of the motions. Its direction, indicated by the broken line, is within about 4° of that assigned in my general investigation of Boss's catalogue. Other determinations generally agree closely. As regards the cause of this polarity, the diagram clearly suggests that it is due to two distinct systems of stars, whose motions overlap to some extent, but are, nevertheless definitely distinguished by the two peaks. I have given elsewhere * diagrams of the theoretical appearance of the equifrequent lines for the ellipsoidal and the two-drift theories. The lines actually found appear to be due to a two-drift distribution; the left-hand drift contains fewer stars than the right-hand one, as former results have indicated. On a unitary theory such as the ellipsoidal hypothesis, there should be no separation into two peaks.

Exactly the same type of figure was obtained in three other investigations of these data, viz.—

1. A first approximation assuming f_1 and h_1 to be simple error-curves

2. Taking f_1 an error-curve and h_1 of the form

$$\{a_0 + a_4(\mu - \mu_0)^4\}e^{-\kappa^2(\mu - \mu_0)^2},$$

and smoothing out rather arbitrarily the wildly oscillating functions which result.

3. As in this investigation, but taking the component of solar motion .56 instead of .43.

The same two peaks and even the same minor irregularities showed themselves in all the investigations.

The third stream or drift makes itself evident in the bulge of the figure towards the solar antapex, and produces the principal disturbance of the symmetry of the figure; this is the more

* "Stellar Distribution and Movements," *Brit. Assoc. Report*, 1911.

remarkable because the third stream is less fully represented in this part of the sky than nearer the galactic plane, and the Orion stars which form an important part of it have been excluded. When I first became aware of this stream, I regarded it as a small additional stream, and anticipated that it would be found to consist of stars more remote than those belonging to the two star-streams. The latter suggestion has been negatived by the work of Professor Kapteyn. According to Dr. Halm's views the bulge is not so much caused by a small extra stream, but is an indication that the right-hand drift in my diagram is itself made up of two systems of about equal importance. This is quite consistent with the appearance of the diagram.

Distribution of the Stars along the Line of Sight.

I had originally intended to derive the function $f(r)$ from Professor Kapteyn's work on the distribution and luminosity of the stars. In *Astron. Jour.*, No. 566 (1904), Table VI., Kapteyn gives provisionally the information that we require; this work has since been amended by him in some points (*Amsterdam Proceedings*, x. p. 626 (1908)), but the results of this later work are not given in a form suitable for our purpose. The emendations affect chiefly the fainter and more distant stars, and do not appear to affect vitally the part of Table VI. which I am about to use. Using the 1904 paper, I find that for stars brighter than $6^m.6$, Kapteyn's figures correspond closely to the formula

$$f_1(\lambda) = \text{const.} \times e^{-\xi(\lambda - \sigma)^2},$$

where

$$\xi = .45$$

and σ corresponds to a parallax of $0''.008$.

But if

$$h_1(\mu) \propto e^{-\kappa^2(\mu - \mu_0)^2},$$

we have seen that

$$\frac{1}{\kappa^2} \text{ must be greater than } \frac{1}{\xi}.$$

Hence $\frac{1}{\kappa} > 1.49$, or, changing to decimal logarithms,

$$\frac{1}{\kappa'} > .65.$$

(If we had taken the lower limit of magnitude to be $7^m.6$, we should have found $\frac{1}{\kappa'} > .62$; thus the indefiniteness of the lower limit of Boss's stars makes little difference.)

Now, the observed values of κ' in Table III. generally fail to satisfy the inequality, and it would be impossible to obtain a solution with this value of $f_1(\lambda)$. Even if, instead of splitting $h_1(\mu)$ into three error-curves, we are content with an approximate

representation by a single error-curve, the corresponding values of $1/\kappa'$ are higher, but still fall below the limit; the values (for sectors containing more than 60 stars) are in fact—

Sector.	335°-5°	5°-35°	35°-65°	65°-95°	95°-125°	125°-155°
$1/\kappa'$	·53	·56	·56	·62	·63	·59

The physical meaning appears to be this. The logarithms of the individual proper motions deviate from the mean value for two causes: (1) because the logarithms of the individual linear motions deviate from the mean, (2) because the logarithms of the distances of the stars deviate from the mean; these two sources of deviation compound like probable errors. Now, the deviations in distance given by $\xi = \cdot 45$ are already more than sufficient to explain the observed differences of proper motion, leaving less than nothing to be attributed to cause (1), although we cannot doubt that that ought to make a large contribution. There is thus a contradiction between our observed data and the Table VI. of *Astron. Jour.* 566.

The contradiction may be partially explained by the fact that we are considering a limited region of the sky of high galactic latitude, whereas Professor Kapteyn's table refers to an average over the whole sky; the number of very distant stars in this region will be very much smaller than the average. It is difficult to make a precise calculation of how this will affect ξ ; but it is hard to believe that this will account for the whole of the discrepancy.*

As I was thus unable to rely on an extraneous determination of distribution of the stars as regards distance, I calculated the functions in the way already described. The function $F(-q)$ has been given in Table IV.; and we can by equation (4) calculate the corresponding distribution of the stars as regards distance from the Sun. The result will refer to the stars of the catalogue, *i.e.* to stars brighter than a limiting magnitude which may be taken to be about $6^m \cdot 6$; this must not be confused with the totally different problem of the absolute density of distribution of stars in different parts of space. The latter problem has a more immediate application to the structure of the stellar universe; but there is a practical interest and importance in learning the distribution of those stars which appear bright to us. The present result is provisional only, because it is a bye-product in an investigation made for a different purpose; and it would have been possible to obtain a superior result, if this had been made my main object.†

* [Added March 9.]—It is interesting to note that in a paper just received Professor Charlier (*Lund Observatory*, "*Meddelanden*" Series 2, No. 8, p. 48) similarly finds himself unable to reconcile Professor Kapteyn's parallaxes with the proper motions of Boss's Catalogue.

† I have now in hand a direct investigation of this kind, strictly limited to stars brighter than $6^m \cdot 0$ (for which Boss's Catalogue is complete) and divided according to spectral type.

The result of the calculation is shown in Table VI. The values of $f_1(\lambda)$ for $\lambda = -6.0, -5.6$, etc., are set down as giving the number of stars between distances for which $\lambda = -6.2$ and $-5.8, -5.8$ and -5.4 , etc. This is correct, neglecting second differences. To obtain the parallaxes in the second column, I proceeded thus:—The unit of distance is that at which the adopted unit of linear velocity will subtend the adopted unit of angular velocity. As regards the former, the component of solar motion in the direction $\theta = 80^\circ$ was taken to be 0.43 unit. But from Campbell's value L.O.B., No. 196, this component $= 19.5 \cos 55^\circ = 11.2$ km./sec. Hence our unit of linear motion $= 26.0$ km./sec. The unit of angular motion was taken to be $0''.001$ per annum. From these it follows that the unit of distance corresponds to a parallax $0''.000182$, and therefore

$$\text{parallax} = 0''.000182 e^{-\lambda}.$$

TABLE VI.

Number of Boss's Stars within given Limits of Distance (Provisional Result) for a Region of high Galactic Latitude.

Limits of λ .	Limits of Parallax.	No. of Stars per 1000.
-6.2	0.0897	20.6
-5.8	0.0601	28.7
-5.4	0.0403	42.9
-5.0	0.0270	81.1
-4.6	0.0181	144.3
-4.2	0.01213	201.4
-3.8	0.00814	206.0
-3.4	0.00545	144.2
-3.0	0.00365	64.7
-2.6	0.00245	23.3
-2.2	0.00164	14.1
-1.8	0.00110	

For an equal density of distribution of the stars the numbers in the third column of the table would form a geometrical progression increasing in the ratio $1 : 3.32$. The much slower rate of increase is mainly due to the increasing proportion of more distant stars falling below the limit of magnitude of the catalogue; there is believed to be also an actual falling off of density.

By interpolation I obtain the following Table:—

TABLE VII.

Parallax.	Percentages of Stars.
"10 - "08	1'0
'08 - '06	1'5
'06 - '04	2'9
'04 - '02	9'5
'02 - '015	8'6
'015 - '010	17'5
'010 - '008	11'7
'008 - '006	14'9
'006 - '004	16'6
'004 - '002	10'0
'002 - '001	2'4

Owing to a possible cumulative error, it would hardly be fair to deduce that the 3·4 per cent. unaccounted for above is at all closely equal to the number of stars with parallax greater than 0"·10 or less than 0"·001.

It is interesting to note that 70 per cent. of the stars have parallaxes between the rather narrow limits 0"·020 and 0"·004.

Tables VI. and VII. rest on rather slender material, and, moreover, depend on the assumption $g(u) = ue^{-(u \cdot 43)^2}$ for the sector $65^\circ - 95^\circ$, which was made somewhat reluctantly. But notwithstanding these drawbacks, they should give a rough idea of the true distribution; and in the present very limited state of our knowledge even a rough approximation may be useful. Further, it seems well to exhibit the result, because it isolates, as it were, all the more doubtful part of our argument. If Table VII. can be shown to be seriously in error, the other results of the paper will need to be modified accordingly; but if this table can be accepted as giving a probable distribution of the stars, then the rest of the results, including the diagram of velocity distribution, follow quite rigorously on the sole assumption that the law of velocities is the same at all distances from the Sun.

A Tentative Explanation of the "Two Star Streams" in Terms of Gravitation. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The following speculations started from reading the admirable summary of our modern knowledge of the stellar universe drawn up by Mr. Eddington for the last meeting of the British Association at Portsmouth. It would perhaps be difficult to point to anything quite new in the summary, but its clear presentation of the situation is essentially suggestive. I quote two important passages.

"It is believed that the great mass of the stars, excluding the Milky Way, are arranged in the form of a lens or bun-shaped system. Our Sun occupies a nearly central position, or at least a position midway between the two flattened surfaces."

"It is convenient for the purposes of explanation and for mathematical analysis to represent this bipolarity in the stars' motions as being the result of two systems of stars having become intermingled."

2. These passages will serve to illustrate an inconsistency to which we have been led. Briefly, our stellar universe is thought of sometimes as one, sometimes as two. Is there any way of removing this discordance? Let us take the first attitude and think of our system as a whole, but as finite in extent. We are led to inquire whether this great mass of stars exerts any attraction on the individuals composing it. If we regard it as a homogeneous sphere, the attraction would be towards the centre, and varying directly as the distance, according to a well-known proposition. It is probably neither homogeneous nor spherical, but as a first rude approximation let us consider the results of this central attraction.

3. The kind of result with which we are most familiar is the description of open orbits round the centre. An extreme form of such motion is a perfectly circular orbit, such as is described by a particle of a solid body. The existence of a definite axis characterising our system, viz., that perpendicular to the plane of the Milky Way, naturally suggests preferential motions round this axis.

4. But there is a wholly different extreme type of motion, in which every star would describe a rectilinear orbit through the centre, with harmonic motion. Any motion intermediate between these extreme types, the circular and the rectilinear, can be regarded as made by combining them, and will have the characteristics of both to a partial extent. Let us therefore consider the result of this second type of motion alone.

5. We could divide all the stars into two groups:

- (A) Those approaching the centre,
- (B) Those leaving the centre.

The directions of motion of (A), as seen on the sky by a stationary observer, would all *converge* to the point representing the direction of the centre. The directions of group (B), on the other hand, would all *diverge* from this point, and if produced far enough the great circles representing them would meet in the opposite point and the application to the "two-drift" theory of stellar movements is at once obvious. But since the solar system is in motion, the observer is not stationary: these directions will be displaced on the sky, according to the laws of "aberration" effects. If we know the Sun's motion we can correct the apparent and obtain the true positions. But when the Sun's motion is deduced only relatively to the mass of the stars, we may get a systematic error. At this point we may recall for consideration the following statement in Mr Eddington's summary:—

"Analysing the data on the two-drift theory, it is found that approximately three-fifths of the stars belong to Drift I and two-fifths to Drift II. Drift I is moving with a velocity of 1.52 towards R.A. $90^{\circ}.8$, Dec. $-14^{\circ}.6$; and Drift II with a velocity of 0.86 towards R.A. $287^{\circ}.8$, Dec. $-64^{\circ}.1$, the velocities being measured in terms of the theoretical unit $1/h$ usually employed in this work. These are the motions relatively to the Sun. *The vertex* or direction of the line of relative motion of the two drifts is R.A. $94^{\circ}.2$, Dec. $+11^{\circ}.9$, very near to the position which Kapteyn originally announced."

The motion of the Sun is practically assumed so as to destroy the aberration effects and bring the apex and anti-apex to the vertex and anti-vertex. But it is only the component of the Sun's motion perpendicular to the vertex which is determined in this way. The component in the line of the vertex has no effect in displacing the vertex, but merely alters the apparent absolute velocities of the two streams, and incidentally the numbers of stars assignable to each. Suppose the two streams were equal: *i.e.* suppose there were just as many stars coming away from the centre as approaching it, and with similar velocities, which is a natural supposition. This would only appear to be the fact to a stationary observer. If we are approaching the centre, we increase the apparent velocity of stars coming from it and add to their number the more sluggish approachers. It will be seen above that Drift I is presumed to be moving faster and to be more numerous. Is this a result of an erroneous value for the Sun's component in the line of vertices?

6. This point is worth re-examination. But it is to be remarked that the figures quoted by Mr. Eddington are by no means irreconcilable with our hypothesis just as they stand. It is not in the least necessary that there should be the same number of stars approaching the centre as leaving it. If the central attraction varies as the distance, all the orbits are described in the same time. The whole system may be regarded as vibrating like an elastic body, expanding at one time and contracting at

another. [This character would still be the same if the general body be an ellipsoid instead of a sphere; the periods along the three axes would be different, but at any moment there would at a given point be a resultant vibrational movement, with a component either towards or from the centre.] Hence there might well be an inequality between the streams.

7. Let us now consider the other extreme form of orbit, the circular. In the neighbourhood of the Sun all the stars, including the Sun himself, would be moving at right angles to the radius vector. If they were all in the same direction we should have a state of things similar to the rotation of a solid body, which it might be impossible to detect from observation alone. But there is no reason to suppose that this is likely. It seems far more likely that they would be moving in both directions, though here again we have no occasion to presume them equal in number. Any excess in one direction would tend to act like a general rotation which may be neglected as probably difficult to detect; and we should thus apparently be left with two nearly equal opposite streams. *But we must remember that the Sun is moving with one of the streams.* Let us consider this one first. Suppose V is the Sun's velocity. The stars outside it are moving quicker and those inside it more slowly: the average translational motion would be null, but there would be an apparent rotation round the Sun. The case would be somewhat similar to that of a rotating solid body when we suddenly bring one point of it, not the centre, to rest: the body will continue to rotate about that point as a new centre.

Now consider the other stream. The apparent translational movement would be $2V$, and the rotation would be in the opposite direction. When we take averages of the whole mass of stars, the rotations would cancel out, and we should be left with half the stars apparently stationary, and half moving with velocity $2V$. We should, in taking the average of all, restore the velocity $-V$ to the Sun, and be left with two streams in opposite directions. *But we should only get one definitely marked apex:* for the stream with average null velocity would refuse to mark out an anti-apex. The apex would be at right angles to the radius vector, not along it, as in the case of radial motions.

8. But in the above elementary analysis we have only considered the stars in the immediate neighbourhood of the Sun, which are moving approximately parallel to his own direction of circulation. Stars at a distance will move in quite different directions, and we must make a more general analysis in order to take them into account. We have compared the circulation of the stars to the rotation of a solid body: a completer analogy would be made up of six separate rotations, two in opposite directions round the axis of x , two round that of y , and two round that of z ; and for the present we will consider all of these equal in magnitude and the same in period. Let the Sun lie on the axis of x , and be circulating round the axis of z in the positive

direction. Now each one of the six rotations can be transferred to the Sun by adding a suitable translation. The rotations alone would thus be the same at the Sun as at the centre and would cancel out, leaving no resultant apparent rotation. So far as they are concerned the stars would appear from the Sun to be moving in all directions indifferently and there would be no trace of an apex. Consider now the necessary translations. The rotations round the axis of x do not contribute translations at all, since the axis passes through the Sun. The rotations round the axis of y contribute two equal and opposite translations parallel to the axis of z ; and those round z two translations parallel to the axis of y . If for a moment we neglect the Sun's own motion, the net result would be a series of translations in all directions parallel to the plane yz , which would tend to intersect on the axis of x , that is, in the direction of the central attraction or its opposite. We should thus get a strengthening of the indications for the apices. Now restore the Sun's motion: its effect on the components parallel to the axis of z would be of the "aberrational" kind, tending to displace the apices in the well-known way. The components parallel to the axis of y would be affected in the manner considered in the last paragraph—one apex would be lost or blurred.

9. In practice the stellar movements no doubt partake of both characters, radial and circulatory, the orbits being ellipses of unknown eccentricity. One vertex will thus be displaced by the composition (in some unknown ratio) of the directions due to the radial and circulatory movements. The other will be displaced less, or not at all, but will be blurred. The best marked apex is that for Drift I, R.A. $90^{\circ}8$, opposite to the solar motion in R.A. about 270° ; and this apex corresponds, as it should, to the solar component round the centre. But the problem of determining the exact direction in space of the attracting centre becomes one of assigning relative values to the Sun's radial and transversal motion, which may not be easy.

10. We may notice in passing a rough analogy for the suggested movements of the stars. Suppose that at any moment we could take stock of all the comets which visit our Sun. There would be some on their way to him and some on their way back into space. Their orbits are ellipses, but approximate to radial motions. The directions of movement of those on their way in would tend generally towards the Sun, of those on their way out towards the anti-Sun.

11. It seems clear that an explanation of the "two star streams" could be framed on these lines, if the observed facts will fit, which can only be ascertained by detailed investigation. But meanwhile there are some other facts of a general nature, of which a possible explanation can be given in terms of this hypothesis. The general conception is that of *vibration*, so that the "initial" state of things recurs at stated intervals (using the word initial in the arbitrary sense of the mathematician). If the

general mass of stars is spheroidal (or ellipsoidal) in shape rather than spherical, the idea of vibration is not destroyed: we get two (or three) fundamental periods instead of one; and even if they are not commensurable, they will be approximately so; consequently there will be periods when the initial configuration recurs approximately. Hence there is nothing unreasonable in our finding a whole group of stars endowed with similar positions and velocities at this moment. If they "happened," so to speak, to be started under uniform conditions, these conditions would tend to recur even after the lapse of ages; meanwhile they may have been backwards and forwards, through or round the centre, many times. A supplement to this suggestion is dealt with at the end of the paper.

12. Again, there is nothing unreasonable in different classes of stars having different average velocities. It is natural to think back to a primitive nebula. Some parts of the nebula would be started near its centre, and for ever describe small orbits; others would start near the boundary and describe large orbits, or, let us say, make large oscillations. The velocities of the former would always be small: those of the latter would be great when they were near the centre. Since our Sun is near the centre at present, we observe these large velocities in stars near us which have fallen in a long way.

13. Again, there is nothing unreasonable in finding that these stars with different velocities have different spectral types, or that the order of their spectral types and velocities should be the same. The physical and chemical constitution of the primitive nebula would tend to be modified from centre to surface, and this might well have an effect on the subsequent history of the stars into which it gradually resolved. To take one line of thought as an illustration: suppose the parts near the centre condensed as a rule into larger masses (owing perhaps to the central condensation), and consequently did not run through their stellar history so rapidly. They would then retain their juvenile appearance longer. These are the stars with small velocities: and hence we might easily have low velocity associated with youthful appearance, without any necessity for supposing that motion was gradually acquired from rest.

The contrast between this explanation and that hitherto presented may be thus expressed.

The type B stars have at present an average velocity of $6\frac{1}{2}$ km./sec., and type M stars 17. It has been presumed that in the distant future the B stars will become M, and in so doing acquire an average velocity 17. On the present view they will retain their present velocity $6\frac{1}{2}$; but the M stars, with velocity 17, will then have reached a type Z, say, perhaps invisible to us. But if we could still see them, the order of velocities would still be that of spectral type.

14. Finally, there is nothing unreasonable in the notion of a particular type of stars expanding outwards in all directions. They

may have been formed at the periphery (say) of the primæval nebula about the same time; have fallen towards the centre together, and may be now together on their way out. Professor Campbell's quantity K may be real and not a systematic error of observation, as he has suggested.

15. Having reviewed the possibilities in a general way, we now proceed to examine the known facts in order to see how far they support the idea of a central attraction, and what information may be gathered concerning the structure and size of the cluster, its fundamental period (or periods) of vibration, the situation of our own Sun with reference to the centre and possibly its orbit. And it is natural to begin with the structure of the cluster itself.

16. One point may first be noticed. It is essential to the hypothesis that the motions should be at any rate partially *radial* in character. If they were strictly radial, there would be an intolerable congestion at the centre. In removing this defect, can we retain orbits sufficiently elongated to mark the apices? We know that the apices are not very well marked, and that in fact it took all Professor Kapteyn's acuteness to discover them, and even now requires care to distinguish them. But it is only elementary precaution to form some estimate, however rough, of the admissible elongations of the orbits.

17. Let us regard the major axis a as given, and consider a particular orbit with minor axis b . The star will at different times be at all distances less than a and greater than b ; but it will pause perceptibly at the extremes. If the space round the centre is divided into thin shells of uniform thickness, the time spent in a shell at the extremes will in fact be indefinitely great compared with that spent in intermediate shells; and in estimating the average density of distribution we can limit our attention to these shells only. Near the end of the minor axis its velocity will be na ; and its coordinates at time t after passing the extremity may be written

$$x = na \cdot t, \quad y = b(1 - \frac{1}{2}n^2t^2).$$

If it be then at distance $b + db$ from the centre, we have

$$(b + db)^2 = b^2(1 - n^2t^2) + n^2a^2t^2$$

or

$$n^2t^2 = 2bdb/(a^2 - b^2).$$

Now, the chance that the star is in the shell between b and $b + db$ varies directly as the time t , and inversely as the volume $4\pi b^2db$. If db is constant, and if we suppose that there are $f(b)$ stars with minor axis b , the average density of distribution at distance b will vary as

$$\frac{f(b)}{b^2} \left(\frac{b}{a^2 - b^2} \right)^{\frac{1}{2}}.$$

Suppose we make this constant: then

$$\begin{aligned} f(b) &\propto b^{\frac{3}{2}}(a^2 - b^2)^{\frac{1}{2}} \\ &\propto a^{\frac{5}{2}} \sin^{\frac{3}{2}} \theta \cos \theta \\ &\propto a^{\frac{5}{2}} \sin 2\theta \cdot \sin^{\frac{1}{2}} \theta \end{aligned}$$

if $b = a \sin \theta$. The values of $f_1(b)$ are thus as in the third column below.

TABLE I.

b/a .	θ .	$f_1(b)$.	$f_2(b)$.	$100 f_3(b)$.
·2	12	·18	·18	3·2
·4	24	·47	·40	·04
·6	37	·74	·56	·01
·8	54	·85	·52	·001
1·0	90	·00	·00	·000

18. There would thus be a fair number of orbits of high eccentricity, even on this hypothesis of uniform distribution. But it is not likely that, on the hypothesis of an isolated cluster of stars, the density would be uniform except perhaps close to the centre. Mr. H. C. Plummer has recently published a valuable research on the distribution of stars in globular clusters (*Mon. Not.*, lxxi. p. 462), showing that the density law found by Schuster for gases

$$\phi(r) = (c^2 + r^2)^{-\frac{5}{2}}$$

fits the observations fairly well. The density at the centre is represented by c^{-5} and at distance c by $c^5/5\cdot7$, so that c is comparable with the sensible dimensions of the cluster. Let us put $c = a$. Then

$$f_2(b) \propto \sin 2\theta \cdot \sin^{\frac{1}{2}} \theta \cdot (1 + \sin^2 \theta)^{-\frac{5}{2}},$$

and is represented in the fourth column of above table. The proportion of much elongated orbits is increased, but is still not very large.

But of course the cluster extends far beyond the point $a = c$. Theoretically it extends to infinity, and the divisor $(c^2 + r^2)^{\frac{5}{2}}$ or $(c^2 + a^2 \sin^2 \theta)^{\frac{5}{2}}$ would then vary nearly as $a^5 \sin^5 \theta$. The expression $f(b)$ would then become nearly

$$\sin^{-\frac{1}{2}} \theta \cos \theta,$$

infinite at the centre and decreasing outwards: *i.e.* the vast majority of orbits would be greatly elongated. But this does not mean that they would all pass close to the centre; for b may be large even though b/a is small. To fix the ideas put $a = 10c$. Then we get $f_3(b)$ as in the fifth column above, multiplied by 100 (the unit is arbitrary) to avoid numerous decimals. Hence the orbits

are highly elliptic, but the minor axes $b = 0.2a = 2c$ are greater than even the major axes of the last case. Such orbits would be likely to furnish apparently parallel streams of stars, and would modify the conception of strictly radial motions materially: they would substitute for the system of straight lines meeting in a point, a system of hollow cylinders, the axes of which met in the point. But the convergence of the axes would still be indicated. To present the matter in another way: if $b = 0.2a$ and we are at a distance comparable with a , a number of orbits will be edgeways to us, and their stars will seem to cross the centre.

19. Let us now consider the structure of the cluster as it would appear to us. We are probably not at the centre, but it will be convenient in the first instance to suppose that we are, and then to make allowance for the effects of our displacement. We will also, in the first instance, assume the distribution spherical, and according to the law quoted by Mr. H. C. Plummer: viz.,

$$\phi(r) = (c^2 + r^2)^{-\frac{5}{2}}.$$

If the individual stars were all of the same intrinsic magnitude, the number within a shell radius r would be

$$N = N_1 \int_0^r \frac{r^2 dr}{(c^2 + r^2)^{\frac{5}{2}}} = N_0 r^3 (c^2 + r^2)^{-\frac{3}{2}}.$$

If the density were constant, we should find $N = N_0 r^3$, leading to the ratio 4 for consecutive magnitudes. But it is well known that this ratio does not hold, and we have now to examine how far this defect can be accounted for by the assumed distribution, which adds the factor $(c^2 + r^2)^{-\frac{3}{2}}$. Let us form the logarithm of the new ratio: viz.,

$$\begin{aligned} \log N_{m+1} - \log N_m &= 3 (\log r_{m+1} - \log r_m) \\ &\quad - \frac{3}{2} \{ \log (c^2 + r_{m+1}^2) - \log (c^2 + r_m^2) \}. \end{aligned}$$

Since $\log r_m = 0.2m$, the first term is 0.6, giving the theoretical ratio 3.98 for uniform distribution.

20. The value of the second term depends on the value adopted for c , which is the distance at which the density has dropped to about one-sixth of that at the centre. At first r is small compared with c , and the second term varies approximately as r^2/c^2 , which increases in the ratio 2.512 for each magnitude.

To fix the ideas put $r/c = 0.1$. Then for all values of r less than this the "star-ratio" is sensibly 3.98; but for the magnitudes fainter we have successively

$$r^2/c^2 = 0.025, \quad 0.063, \quad 0.158, \quad 0.398 \text{ etc.},$$

so that the approximation is no longer valid. When r is large compared with c the value of the second term tends to

$$-\frac{3}{2} (\log r_{m+1}^2 - \log r_m^2) = -0.6,$$

so that

$$\log N_{m+1} - \log N_m = 0,$$

and the addition of stars ceases.

21. It will be seen that the interesting part of the series for $\log N_m$ is very short. We need not trouble about the portion of sensibly uniform distribution. Beginning with $r^2/c^2 = .10$, we get for successive magnitudes the first column of the following table; and the process will be understood from the headings. The series may be continued indefinitely forwards as constant, and indefinitely backwards as a geometrical progression with ratio sensibly 3.98.

TABLE II.

$(1+r^2/c^2)$.	Log.	Diff.	$\frac{1}{2}$ diff.	$\Delta \log N$.	Log N.	N.
1.10	.0414				.000	1.0
1.25	.0969	.0555	.083	.517	.517	3.3
1.63	.2122	.1153	.172	.428	.945	8.8
2.58	.4116	.1994	.300	.300	1.245	17.6
4.98	.6972	.2856	.428	.172	1.417	26.1
11.00	1.0414	.3442	.516	.084	1.501	31.7
26.00	1.4150	.3736	.561	.039	1.540	34.7
64.00	1.806	.391	.586	.014	1.554	35.8
159.00	2.201	.395	.593	.007	1.561	36.4
399.00	2.601	.400	.600	.000	1.561	36.4

22. Now it is clear that this state of things does not correspond to our universe as we know it. The ratio is not steadily 4 for a number of magnitudes, with a drop to unity within half a dozen magnitudes: it is steadily less than 4 for a considerable range of magnitudes. According to Kapteyn (*Gron. Pub.*, No. 18), the average increase in $\log N$ for the first ten magnitudes is 0.51, corresponding to a star-ratio of 3.24.

But we have omitted one essential fact: the stars are not all of the same intrinsic brilliancy, they vary enormously. As examples let us take, first, the 17 nearest stars quoted (from the Astronomer Royal) by Mr. Eddington in his B.A. report. Their magnitudes are

-1.6	3.6	5.6	8.8
0.3	3.6	7.6	8.9
0.5	4.7	8.2	9.2
0.9	4.8	8.3	9.2
			9.3

The striking fact is the increase in number of the intrinsically fainter stars, which would be enhanced if we made allowance for parallax, because the chief anomaly, α Centauri, now credited as

0.3, would be 2 or 3 magnitudes fainter. Considering that the brightest stars have been most carefully examined, it seems probable that this excess of intrinsically faint stars is under-represented. There may, for instance, be many eleventh and twelfth magnitude stars equally close.

Secondly, take the intrinsic magnitudes of the type A stars recently calculated by Mr. H. C. Plummer on the lines sketched in *Mon. Not.*, lxxii. p. 170. [I am indebted to him for the results in MS.] They are thus grouped *when reduced to parallax* 0".01. [This is about thirty times more distant than the above group; and hence the magnitudes all appear about 7 less.]

< 0	- 1	- 2	- 3	- 4	- 5	- 6	- 7	- 8	- 9	- 10	- 11	- 12	- 13
3	2	0	5	3	22	17	17	20	10	5	3	3	1

Here the selection has been made, not by similar distance, but by similar apparent brightness (the spectroscopic observations being limited to fairly bright stars) and type.

23. We will now present the problem from the other end. How must the number of intrinsically fainter stars be regulated in order that the "star-ratio" may have the observed value 3.16 ($\log = 0.50$)? We will first tabulate a hypothesis, empirically constructed, and then consider its justification.

In Table III. the column headed N_0 is quoted from Table II. We may imagine it continued upwards in a ratio $1/3.98$. The next column to the right is three times the first moved one place lower down; the next again three times the second, and so on. Each column represents a separate system of stars of the same intrinsic brightness, distributed according to the law formulated above. Above the initial term (corresponding to the first term of N_0), the numbers in each column decrease in the constant ratio 3.98. To the left of the column N_1 the numbers are $\frac{1}{3}$ of N_1 moved one place up. They soon become constant and then diminish in successive columns as a G.P. whose sum is $1/(1 - \frac{1}{3}) = \frac{3}{2}$, the initial term. The top row to the right of N_0 is a G.P. with ratio 0.75, the sum of which is four times the first term. In the second row the G.P. begins with N_1 , in the third with N_2 , and so on. It is easy to form the sums of the rows to infinity in both directions, and the results are given in the column headed N . They increase in the simple ratio 3.0, and it is easy to see that each row is three times the one above it moved one place to the right. If we had adopted Kapteyn's ratio 3.24, we could have reproduced it similarly in column N ; but the simple value 3 makes the formation more obvious, and is near enough for our purpose.

Nothing is yet assumed about the difference in intrinsic magnitude between consecutive columns: it may be one magnitude or two, or any other number: it can be chosen later so as to fit observed facts. Indeed, we have nothing to show as yet whether successive columns are increasing or decreasing in intrinsic brightness; but we will commence by assuming that the first

column represents the intrinsically brightest stars, the others successively fainter sets.

TABLE III.

N_{-3}	N_{-2}	N_{-1}	N_0	N_1	N_2	N_3	N_4	N	App. Mag.
0.7	1.0	1.1	1.0	0.8	0.6	0.4	0.3 ...	7.3	0
1.0	2.0	2.9	3.3	3.0	2.3	1.7	1.3 ...	21.8	1
1.2	2.9	5.9	8.8	9.9	9.0	6.8	5.1 ...	65.3	2
1.3	3.5	8.7	17.6	26.4	29.7	27.0	20.3 ...	195	3
1.3	3.9	10.6	26.1	52.8	79.2	89.1	81.0 ...	585	4
1.4	4.0	11.6	31.7	78.3	158.4	238	267 ...	1755	.5
1.4	4.1	11.9	34.7	95	235	475	714 ...	5265	6
1.4	4.1	12.2	35.8	104	275	705	1425 ...	etc.	7
1.4	4.1	12.2	36.4	107	312	825	2115 ...	etc.	
1.4	4.1	12.2	36.4	109	321	936	2475 ...	etc.	

24. It is highly improbable that such systems can be superposed without mutual disturbance, since their extent is so different. The compact systems are concentrated near the middle, and would thus exert a central attraction on the diffused systems, varying more nearly as the inverse square than as the direct distance. On the other hand, these diffused systems would have little influence on the distribution of the central stars, being mainly outside them. But the suggested arrangement may give us a qualitative idea of the actual facts.

25. Now let us consider some consequences of this law of distribution.

Firstly, let us consider Mr. Plummer's group of intrinsic magnitudes. They are selected, firstly, as stars brighter than sixth magnitude by the necessities of the spectroscope; secondly, as type A stars.

We are scarcely in a position as yet to take the second condition into account, and must neglect it. We should expect then to find a distribution like the fifth or sixth row in Table III. [Remember that N itself is the total number of stars brighter than a given magnitude, so that no summation is necessary; further, that our starting point is somewhat arbitrary.]

Now, his total numbers up to and including the magnitude shown in the first column below are as given in the second column under heading n ; and then $\log n$ is formed. The horizontal rows of Table III. are essentially similar, but start at different points. If we take the fifth and form by summation the series 2.0 (sum of all columns up to and including the first), 5.9, 16.5, etc., and subtract 0.20 from the logarithms of these numbers, we get the column headed C , which agrees fairly well with O , as shown in the column $O-C$. Converting the calculated $\log n$ back into numbers, we get the column n , for comparison with n . This comparison suggests, therefore, that the interval between con-

secutive columns in Table III. is about 2 magnitudes of intrinsic brightness.

Intrinsic Mag. H.C.P.	$n =$ Total Stars.	Log n O.	5th row C.	O - C.	n_c
- 1.0	1	0.00	0.10	- 0.10	1.2
1.0	5	0.70	0.57	+ .13	3.8
3.0	10	1.00	1.02	- .02	10.5
5.0	36	1.56	1.43	+ .13	27
7.0	70	1.84	1.78	+ .06	60
9.0	100	2.00	2.04	- .04	110
11.0	108	2.03	2.22	- .19	167

26. We can now estimate the relative density of the systems. Each of them contains just 3 times as many stars as the one before it, but they are diffused over different volumes. Let us represent the distances from the centre of the stars in columns N_1 and N_2 by

$$\begin{array}{ll} r_1 & r_2 \\ r_1 s & r_2 s \\ r_1 s^2 & r_2 s^2 \\ \text{etc.} & \text{etc.,} \end{array}$$

where s is the ratio corresponding to 1 magnitude $= 10^{.2} = 1.58$. Then, if the intrinsic brilliancy I_1 of the first column is 2 magnitudes brighter than I_2 , we have

$$r_1^2/r_2^2 = 6.3,$$

or

$$r_1 = 2.5 r_2.$$

Now, corresponding distances for the two systems are r_1 and $r_2 s$, so that the ratio of c_1 to c_2 is

$$c_1/c_2 = r_1/sr_2 = 1.58.$$

Hence N_1 is spread over $(1.58)^3 = 4$ times the volume occupied by N_2 , and has only one-third the total number of stars. The central density must be one-twelfth that of N_2 . We thus get the interesting result that the intrinsically bright stars are far more widely diffused than the faint. Perhaps we might have expected this, for in the contrary case there would have been a group of very bright stars at the centre which could scarcely have escaped observation.

27. At the centre of the system, then, we should expect, within a given sphere, a few very bright stars, 12 times as many of the next system 2 magnitudes intrinsically fainter, 12 times as many again of the next, and so on, continually increasing. This state of things is grossly unlike anything we know, although

"the 17 nearest stars" show a decided increase towards the ninth magnitude.

28. But then we are probably not at the centre of the system. We must examine the effect of the Sun's excentric position.

Suppose the Sun at the distance a from the centre. The density in its neighbourhood is

$$\phi(r) = (c^2 + r^2)^{-\frac{5}{2}} = (c^2 + a^2)^{-\frac{5}{2}} \text{ nearly.}$$

If we wish to find the number of stars within a sphere radius ρ round the Sun as centre, we must evaluate a rather troublesome integral, since

$$r^2 = a^2 - 2a\rho \cos \theta + \rho^2.$$

But it will be sufficient at present to note that when r is small the density is nearly constant as above, and the total number of stars thus varies as

$$r^3 / (c^2 + a^2)^{\frac{5}{2}},$$

and when r is large compared with a the numbers are much the same as for the centre, viz.,

$$r^3 / c^2 (c^2 + r^2)^{\frac{5}{2}}.$$

29. Near the Sun, then, the densities of the superposed systems compare with those at the centre in the ratio $c^5 / (c^2 + a^2)^{\frac{5}{2}}$. When c is large compared with a this approaches unity; when c is small, it is very small. Starting with $c=a$ and giving c values corresponding to 2 magnitudes fainter each time (*i.e.* diminishing it in the ratio 1.58 each time), we get the following values for the log ratio:—

System.	c/a .	Log $c^5 / (c^2 + a^2)^{\frac{5}{2}}$.	Log Central Density.	Sum.	Density at Sun.
0	1.00	9.25	0.75	0.00	1.0
1	0.63	8.64	1.83	0.47	3.0
2	0.40	7.84	2.91	0.75	5.6
3	0.25	6.94	3.99	0.93	8.5
4	0.16	5.98	5.07	1.05	11.2
5	0.10	4.99	6.15	1.14	13.8
6	0.063	4.00	7.23	1.23	17.0

Thus the rapid increase in the number of intrinsically faint stars, so conspicuous near the centre of the system, only holds in the neighbourhood of our Sun up to a certain point; then it begins to drop to a very gentle increase, the limit of which has the logarithm

$$\log 12 - 5 \times .200 = 0.079,$$

so that the ratio itself is 1.2 instead of 12. Both ratios depend

on the assumed difference of intrinsic magnitude between the columns of Table III., which we have adopted as 2 magnitudes. Had we made it $2\frac{1}{2}$ magnitudes, we should have had

$$r_1 = 3.2 r_2, \quad c_1 = 2c_2.$$

Central densities increase in ratio 24. At Sun, $\log(\text{ratio}) = \log 24 - 5 \times 300 = -0.10$, *i.e.*, the number of faint stars per unit volume would *decrease*, which is contrary to the evidence of the "17 nearest stars," and other evidence to be presented later. Hence 2 magnitudes is probably not far from the correct value.

30. Now let us turn to the "17 nearest stars." Four presentations of the material are given below, grouping in intervals of 2 magnitudes, but starting at different initial points, to avoid accidental bias:—

Limits of Mag.	No.	Limits of Mag.	No.	Limits.	No.	Limits.	No.
-3.5 to -1.6	1	-3.0 to -1.1	1	-2.5 to -0.6	1	-2.0 to -0.1	1
1.5 to 0.4	1	1.0 to 0.9	3	0.5 to 1.4	3	0.0 to 1.9	3
0.5 to 2.4	2	1.0 to 2.9	0	1.5 to 3.4	0	2.0 to 3.9	2
2.5 to 4.4	2	3.0 to 4.9	4	3.5 to 5.4	4	4.0 to 5.9	3
4.5 to 6.4	3	5.0 to 6.9	1	5.5 to 7.4	1	6.0 to 7.9	1
6.5 to 8.4	3	7.0 to 8.9	5	7.5 to 9.4	8	8.0 to 9.9	7
8.5 to 10.4	5	9.0 to 10.9	3	9.5 to 11.4	—	10.0 to 11.9	—

All four series indicated a steady increase, masked by

- (1) accidental irregularities,
- (2) failure of faint stars.

The failure of faint stars may be simply due to the fact that they have not been detected, or to a physical fact. The accumulation of faint stars near the centre is a point that must be examined, and we will return to it. But assuming the series complete as far as they go, and assuming that they represent GP's with ratio q : then, in the first two cases, the ratio of the last three terms to the first three should be q^4 . Hence

$$\begin{array}{ll} \text{the first} & \text{gives } \log q = (\log 11 - \log 4)/4 = 0.11, \\ \text{,, second} & \text{,, } \log q = (\log 9 - \log 4)/4 = 0.09, \\ \text{,, third} & \text{,, } \log q = (\log 13 - \log 4)/3 = 0.17, \\ \text{,, fourth} & \text{,, } \log q = (\log 11 - \log 6)/3 = 0.09, \end{array}$$

The mean for $\log q$ is thus 0.11, which is greater than the limiting value 0.079 found in § 29. Hence, if we may assume that these numbers correspond to the limiting slope, the intervals between the columns of Table III. are rather less than 2 magnitudes.

31. Let us repeat the calculation of § 29 in general terms.

Let us put

$$\log I_1 - \log I_2 = 2i = 2 (\log r_1 - \log r_2).$$

Then

$$\begin{aligned} \log c_1 - \log c_2 &= \log r_1 - \log r_2 - \log s \\ &= i - 0.2. \end{aligned}$$

Now $N_2 = 3N_1$, and the volumes are in the ratio c_1^3/c_2^3 . Hence, if D be the central density,

$$\begin{aligned} \log D_2 - \log D_1 &= \log 3 + 3(i - 0.2) \\ &= 3i - 0.12. \end{aligned}$$

At a distance a , large compared with c , the density d is reduced in the ratio c^5 . Hence

$$\begin{aligned} \log d_2 - \log d_1 &= 3i - 0.123 - 5(i - 0.2) \\ &= 0.877 - 2i, \end{aligned}$$

decreasing as i increases, as already found. When $2i = 0.8$, corresponding to a difference of 2 magnitudes in intrinsic brilliancy, we have

$$\log d_2 - \log d_1 = 0.077.$$

If we put

$$\log d_2 - \log d_1 = 0.11 = \log q$$

(as indicated by the last paragraph), then

$$2i = 0.767;$$

and since it seems highly probable that there are other faint stars near us whose parallaxes have not been determined, we may be prepared to find an even smaller value for $2i$. We may tabulate the possibilities as follows:—

Let u be the *limiting ratio of density per magnitude* in the neighbourhood of the Sun. Then, since

$$\log d_2 - \log d_1 = 0.877 - 2i$$

represents the log ratio for $2i$ magnitudes,

$$2i \log u = 0.877 - 2i$$

or

$$\log u = 0.877/2i - 1.$$

The 17 nearest stars suffice to show that u is greater than unity, so that $\log u$ is a small positive quantity. Giving it the values below, we can calculate $2i$, and express the result in magnitudes.

$\log u =$	0.10	0.15	0.20	0.25	0.30	0.35	0.40
$2i =$	0.80	0.76	0.73	0.70	0.67	0.65	0.63
Interval in mags.	2.00	1.90	1.83	1.75	1.68	1.63	1.58

32. As a third test case we will now take the Oxford proper motions of faint stars. These were found from the examination of 183 plates containing 33,000 stars, and no selection of magnitudes was made, all stars, bright or faint, coming equally under scrutiny. Mr. Bellamy finds (*Mon. Not.*, lxxii. p. 83) for the numbers of stars showing more than 20" per century P.M. up to and including the magnitude specified—

To 7·9	8·9	9·9	10·9	12·0
21	29	45	69	89

and between 15" and 20"—

14	26	51	85	117
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Can we take these specified proper motions as corresponding to specified parallaxes? There is an obvious assumption involved—that the intrinsic motions are the same. For the present, let us make this assumption. Then the differences between logarithms for the two series are

·14	·19	·19	·11	Mean ·16
·27	·29	·22	·14	Mean ·23

Hence $\log u$ is almost certainly less than ·29 and greater than ·11. Let us take $\log u = \cdot 20$, $i = \cdot 36$.

Interval in magnitudes in Table III. = 1·83

$$\log (\text{ratio of central densities}) = 3i - \cdot 12 = \cdot 96 = \log 9\cdot 1$$

$$\log c_1/c_2 = i - \cdot 2 = \cdot 16 = \log 1\cdot 44$$

$$(\log \text{limiting ratio of densities at the Sun}) = \cdot 877 - 2i = \cdot 15 = \log 1\cdot 4.$$

33. Now let us return to the previous examples. First, as regards the 17 nearest stars: since $2 \log u = \cdot 40$, it seems clear either that one or two of the very bright stars, such as α Centauri and Sirius, must be regarded as exceptional in the volume considered, or that a number of faint stars (mags. 7–10) with considerable parallaxes have not yet been detected. Neither of these suppositions is unreasonable.

34. As regards Mr. Plummer's figures, our assumption has still to be corrected for the fact that our Sun is not at the centre of the system. The general effect of this is that the intrinsically fainter stars, instead of increasing in the ratio 9·1, as they do at the centre, only increase in a ratio tending to the limit 1·4. This has the effect practically of suppressing the columns of Table III. on the right after a certain point. This point may not be easy to determine with our present knowledge, but we can see at a glance what its general effect will be. The columns to the right increase in the ratio 4 for a considerable distance. Their suppression, therefore, will diminish the ratio 3 to 1 still further. Now, since it was our intention to suit the case of the Sun (as a

view point), and not that of the centre of the system, we ought to have adopted a rather higher ratio, in view of this reduction to come. But this correction must be made at a future time.

Mr. Plummer's results can be arranged as follows, in steps of 1.85 mags.

Intrinsic Mag. (H.C.P.).	Total Stars.	Log n =0.	5th row C.	O-C.
to -0.4	2	0.30	0.10	+0.20
to 1.4	5	0.70	0.67	+0.03
to 3.2	11	1.04	1.02	+0.02
to 5.1	37	1.57	1.43	+0.14
to 7.0	70	1.85	1.78	+0.07
to 8.8	100	2.00	2.04	-0.04
All	112	2.05	2.22	-0.17

It is clear that if we begin suppressing the columns on the right at the proper point, we can diminish the last terms in C suitably and get a very good accordance. But it would perhaps be laying too much stress on the material to carry the investigation further at this point.

35. Let us now examine the central condensation. It is possible that the structure arrived at for the cluster will give so great a brightness for the centre, owing to accumulation of stars (which would have been still more striking if the first idea of concentration of bright stars had been justified), as to be inadmissible. Let the Sun be at distance a from the centre as before, and let us first consider a single system with characteristic c . The apparent brightness of a small unit angular area in the direction of the centre as seen from the Sun is proportional to

$$\int_0^a \frac{I.D.c^5 \rho^2 d\rho}{\rho^2 [c^2 + (a-\rho)^2]^{\frac{5}{2}}} = \frac{ac I.D.(2a^2 + 3c^2)}{3(a^2 + c^2)^{\frac{3}{2}}},$$

where I is the intrinsic brilliancy and D the number of stars per unit volume at the centre. This portion is contributed by the stars from observer to centre. But there is also the portion beyond the centre which contributes

$$\int_a^\infty \frac{I.D.c^5 d\rho}{[c^2 + (\rho - a)^2]^{\frac{5}{2}}} = \int_0^\infty \frac{I.D.c^5 l}{(c^2 + r^2)^{\frac{5}{2}}} = \frac{2}{3} I.D.c,$$

and the whole illumination is therefore

$$\frac{1}{3} I.D.c \left[2 + \frac{a(2a^2 + 3c^2)}{(a^2 + c^2)^{\frac{3}{2}}} \right].$$

For comparison let us take the brightness in the opposite direction; and we find

$$\frac{1}{3} I.D.c \left[2 - \frac{a(2a^2 + 3c^2)}{(a^2 + c^2)^{\frac{3}{2}}} \right].$$

The second term in each bracket is zero when $a=0$, *i.e.*, for an observer at the centre the brightness may be called 2. As he recedes the apparent brightness of the centre *increases*, tending to the value 4, the defect from 4 being the brightness in the opposite direction. The fact that the brightness actually increases as we recede would follow with almost any reasonable law of distribution, but I do not remember to have seen it noticed before.

36. Another important point calling for notice is the possibly too great accumulation of *mass* at the centre. The number of stars per unit volume, D , continually increases from one of our adopted systems to the next fainter in the ratio 9:1: the intrinsic brightness I meanwhile falls off in the ratio not quite 6. Hence DI increases continually. But it must be remembered that the effective volume of each system continually diminishes as the cube of the linear dimension, characterised by the quantity c ; and $c_1^3/c_2^3=3$. Hence the total effective brilliancy of each system falls off in the ratio $9/3 \times 5 = 3/5$ about, and the sum remains finite. The same will be almost certainly true for the total effective mass; for though there are some cases, especially among double stars, where greater mass seems to be associated with smaller brilliancy, we can scarcely suppose this to be a general law. It seems even probable that as the brilliancy, varying with the surface, falls off, the mass will fall off in an even greater ratio. But there will in any case be an accumulation of mass at the centre, which would make the law of attraction more nearly as the inverse square than as the direct distance. This suggests a totally different explanation of the progression in mean velocity from that of § 12. The orbits would be like those of comets, with large velocities near the centre and small ones at a distance. The bright stars are, as we have seen, those which travel to great distances, and their average velocities are small. The faint stars are near the centre with the rapid circulation of the planets. The notion of vibration or isochronism of any kind disappears altogether; and the law of distribution for the bright stars must be revised, to take account of this central attraction.

37. Here it will be convenient to conclude this first attempt to account for known phenomena of stellar distribution and movement by means of a central attraction. As I began by acknowledgment of indebtedness to Mr. Eddington for starting a train of thought, so I must conclude by similar acknowledgments to Mr. H. C. Plummer, whose researches on globular clusters and on the distribution of intrinsic magnitudes were indispensable for comparing the hypothesis with fact.

38. But there is another direction in which the main idea of *radial* movements as opposed to circulatory gives a new view of the possibilities: *viz.*, the early history of the stellar system of which our Sun is one member, and of other similar stellar systems. Let us imagine, as Laplace did, that we start with an immense mass of nebular matter; but let us eliminate, or reduce greatly in significance, his idea of a general angular velocity. It is this idea

of *rotation*, to which he attached so much importance, which has obscured a possible view of the sequences which seems to fit known facts much better. Let us for the moment neglect circulatory movement *altogether*. We will presently return to it and make due allowances; but first let us consider the diffused matter of the nebula at rest, or endowed with infinitesimal motion in all its parts, subjected only to its own gravitation, which acts approximately as an attraction to a central point. All the separate parts of the nebula will proceed to fall in towards the centre, and the nebula will contract. The condensation will be symmetrical, slow at first and increasing rapidly, until it culminates in a rush or crush to the centre with considerable speed. The nebula, however diffuse at first, will become so compact ultimately that its particles clash and develop heat and light, converting some of the energy of motion into heat. Indeed, if all the motions are strictly radial, we should reach a state of things of an explosive character which would shoot the particles back again outwards with such velocities that the original diffuse nebula would reform in the same time as it had taken to condense; except that, as some of the energy of motion had been lost, the original extent would not be quite recovered. Allowing for this, after expanding to a stationary state, it would again start on a career of condensation, resulting in another explosion, and so on. Explosions would thus recur at intervals, and on each occasion a distant spectator would record the appearance of a "new star" in the heavens. Does this seem fanciful? We have already seen it happen twice before our eyes. In 1901 Nova Persei blazed up, and illuminated for us its own contracting nebula, of which we had no previous knowledge. The central condensation suddenly became intolerable, though much of the nebula was still widely diffused: we saw it only by the illumination, reflected to us months afterwards, of the central explosion, which may well have been the first to take place in that particular nebula. But the case of Nova Andromedæ was different: there central condensation had already taken place in the past, and some of its results had produced a steadily shining nebula. But there had also been expansion outwards, followed by a new contraction, which culminated in the central condensation of 1885. Some day Nova Persei will again explode; and some day Tycho Brahe's star of 1572 will recur.

39. One point requires further consideration before we proceed to re-introduce the neglected circulatory motions. What is the law of attraction to the centre? Initially we may suppose the nebula a homogeneous sphere for simplicity: the attraction at any point will vary as the distance r . If this law were maintained, every particle would be due at the centre at the same moment. But the law cannot persist. It takes the form M/r inside a homogeneous sphere, because as a particle moves further inwards it loses the attraction of any shell outside it, and it is continually putting new shells outside itself. But when the nebula contracts, as a whole, no new shells are ever put outside the particle:

the attracting mass remains constantly the same as it was at first, viz., that of the sphere on whose surface it was and continues to be. Hence, if a particle starts at distance r_0 from the centre, the equation of motion is simply

$$\ddot{r} = -Mr_0^3/r^2.$$

Integrating once,

$$\dot{r}^2 = 2Mr_0^2(r_0 - r)/r.$$

Putting

$$r = r_0 \cos^2 \theta,$$

r_0 disappears from the equation, and the particle is due at the centre after a time

$$t = \sqrt{\frac{\pi}{8M}}$$

the same for all distances. Hence there is a real period of vibration *so far as the radial motions alone are concerned*, or rather of alternate expansion and contraction (see § 11).

40. We will now return to the neglected circulatory motions. There must of course be such motions, but the idea already used in § 8 must also be brought in here. Laplace had the idea of a single rotation, or circulation, and this has brought troubles, *e.g.*, we look for a sufficient angular momentum to have thrown off a ring, and find none adequate. But a ring may have two angular momentums in opposite directions, like the two opposite rotations of § 8. If it is made up of stars or particles at great distances apart, one can circulate as freely through the other as we suppose our "two star streams" to move rectilineally. Transverse velocities in either direction are enough to keep the ring extended. There is no need to suppose them in one direction alone. For instance, until the spectroscopic observation of Saturn's ring gave us evidence that the particles were all travelling in the same direction, there was no need to presume this. We might have found as great a surprise in store for us as Phœbe provided. Of course the question of stability arises; and very probably the state of things is unstable: the ring would be likely to break up into fragments. But this is just what we want, and what the hypothesis of a single circulation in one direction only has hitherto failed to provide.

41. To return, however, to the main point. Consider a particle of the original nebula. We have supposed it to start from absolute rest, but it will probably have some fortuitous motion, however small. If this happens to be directly in the direction of the centre, it will accelerate the arrival of the particle at the centre: if away from it, it will retard it. In general there will be a component in the radial direction either positive or negative, and these alone will modify the necessity for the whole nebula arriving at the centre simultaneously. Hence from these alone

we can understand why the outburst took place in Nova Persei, while much of the nebula was still far from the centre.

42. But there will also be transverse components. These will set up circulations in both directions. The transverse velocity may not provide sufficient centrifugal inertia at first, but there will come a time when it suffices to arrest further progress inwards, and the particle will describe an orbit like that of a comet. But since the initial velocities are small, these comets will tend to have small perihelion distances, and there will still be the clash at the centre. Different particles will have different experiences; some will extricate themselves with comparative ease and pursue orbits only slightly modified by the collision; but others will collide and possibly coalesce. It would seem to be the time for the formation of stars—the welding together of hitherto separate particles into a single mass. If the original nebula were spherical, the formation would tend to remain symmetrical round a centre; and a globular cluster would ultimately result. The stars would not necessarily all be formed at the same time; there would be a series of recurring "explosions," and each time one or more (probably many) stars would be formed. We thus see a new possibility of explaining the relation between the age of a star and its velocity. The oldest stars would be the product of early "explosions," which would be the most violent, and hence originate the greatest velocities. Each time there is an "explosion," some of the primæval energy is converted into heat and light, and the next will be less violent.

43. But it will be tolerably clear that many other facts can be brought into relation to this hypothesis—for instance, the flatness of the nebula in Andromeda. It is only necessary to suppose that the original nebula was not spherical, and the general case can be represented by an ellipsoid. The forces in different directions are then Ax , By , Cz , obeying the same law, but leading to different periods. The condensation along the shortest axis would occur first. At a particular moment the nebula would therefore be highly compressed in one direction and extended in the others. But it would take too long to follow the further history at present.

The Nebula H.I.159 Cassiopeiæ. By Mrs. Isaac Roberts.
(Plate 3.)

(Communicated by A. R. Hinks.)

In the Isaac Roberts' Collection of Celestial Photographs the nebula H.I.159 (h 71, G.C. 158, N.G.C. 278) is found on the two reflector plates:

I.R. 1727, Nov. 18, 1894,
Exp.: 30^m (from 1^h 30^m to 2^h 0^m sidereal time)
Crowborough.
3197, Jan. 9, 1904,
90^m (from 0^h 58^m to 2^h 28^m sidereal time),

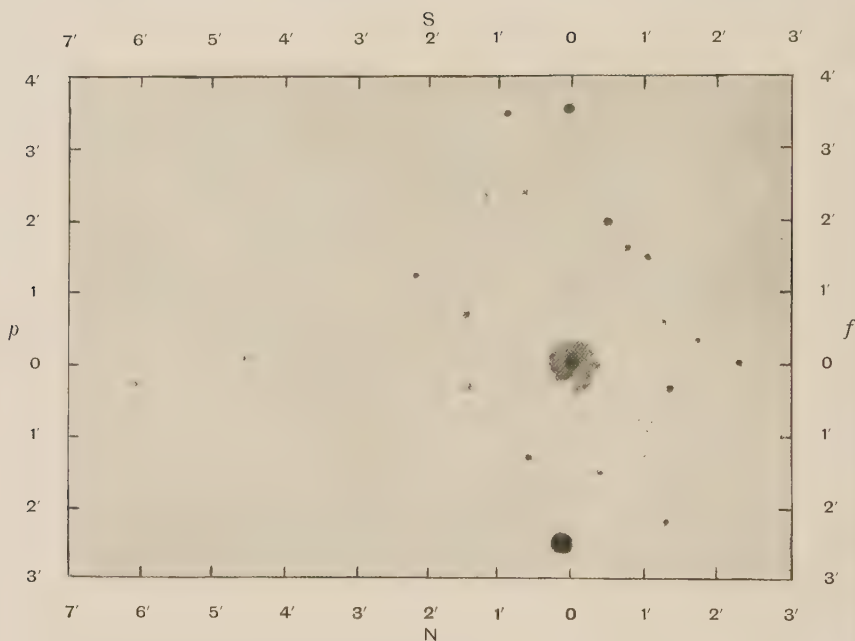
and on the plate:

I.R. 1539 (obtained with the 5-inch lens simultaneously with 3197), where it appears like a small nebulous star.

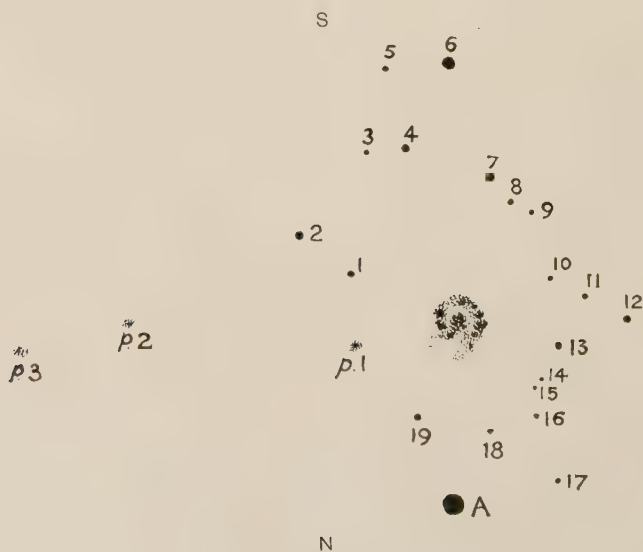
In Isaac Roberts' Register of Photographic Plates the description of the nebula is as follows, the words abridged in the text being here written in full:

- "1727. Nov. 18, 1894: Nebula H.I.159, Cassiopeiæ; R.A. 0^h 46^m; Decl. 46° 59'; exp.: 30^m; sky clear; stars round and crowded on plate. Nebula a dense circle of nebulosity with a projection on the north side; a small stellar nucleus with well-defined margin; six star-like condensations involved in the outer rim of the nebulosity; like a periwinkle; N.G.C. 278. Enlarge for R.A.S. See 3197.
3197. Exp.: 90^m; sky clear, stars round, and crowded on plate; nebula as described on plate 1727 but denser; nebula a spiral with condensations.
1539. 5-inch lens."

Further Description.—The photographs show the nebula to be a left-hand spiral, which is seen full face; it is convex in its *sp*, *s*, and *f* regions, divergent toward the north. There is but one spiral visible, which extends over 60" from north to south and over 50" from *p* to *f*. The spiral converges to its star-like centre, winding twice round it in the northern region of the nebula, thus determining an interspace less dense with nebulosity which contrasts with the denser regions *sp*. Involved in the spiral, and very near its outer margin, are the nebulous condensations C 1, 2, 3, 4, 5, 6, 7, 8, 9, the latter being farthest from the nucleus (see diagram). On I.R. 3917 the nucleus and the nebulous condensations are denser, more diffused, and less well defined than on I.R. 1727, where the background contrasts with the faint condensations; the measures were more difficult on the plate of longer exposure. On the diagram the condensations



DRAWING OF THE NEBULA H.I.159 CASSIOPEIÆ, AND OF THE FAINT STARS IN THE VICINITY OF THE NEBULA.



KEY MAP.

marked by a cross (+) refer to plate I.R. 1727; those marked by a black dot refer to I.R. 3197. Several condensations on this plate appeared with multiple nuclei, due to the long exposure; they were measured separately. The positions of the central nucleus of the nebula on I.R. 1727 and I.R. 3197 are made to coincide on the diagram.

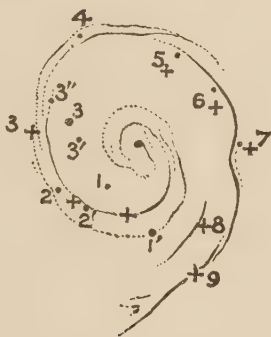


Diagram of Stars within the Nebula.

Measures.—The measures were made by me with the Isaac Roberts' Stellar Pantograver, described in *M.N.* 1888.* Each plate was first punctured at its centre and then placed on the circular stage E (position circle) of the instrument and carefully centred. With plate I.R. 1727, which was measured first, star A was at the zero point of the screws H and G; and the meridian through star A oriented *n* and *s*. With plate I.R. 3197 the puncture of the plate was at the zero points of the screws and the meridian through the puncture oriented *n* and *s*. The plate was firmly fixed on the stage and the position circle firmly clamped. All the measures were then made with the screws H and G in the first position of the circle at 0°. After completion of the first series of measures, the circle was rotated through 180° and clamped, and a second series of measures were made with the screws H and G, the zero point and the object being bisected alternately with the threads of the double micrometer MM. The readings of the divided heads of the screws H and G were inscribed and the differences taken. The means of the measures of the two series were then reduced into standard rectangular co-ordinates, which were converted into R.A. and Decl.

Reduction of Measures.—The method of reduction employed is that in use at the Paris Observatory for the plates of the Astrophotographic Catalogue. The four constants and the concluded R.A. and Decl. of the centre were determined for each

* *M.N.* 1888, pp. 5-13. "On an Instrument for Measuring the Positions and Magnitudes of Stars on Photographs and for Engraving them upon Metal Plates, with Illustrations of the Method of Using the Instrument," by Isaac Roberts.

plate by M. Pourteau of the "Bureau des Calculs de l'Observatoire de Paris," who also reduced the measures of plate I.R. 3197, which was studied to test the method of measurement and reduction employed for I.R. 1727.

Tables.—The objects measured on the plates I.R. 1727, 3197, and the concluded position R.A. and Decl. given in the tables annexed, comprise the standard stars, the nebula H.I. 159, nucleus and nebulous condensations, and the faint stars in the vicinity of the nebula, which were measured with respect to the nucleus and also with respect to the bright star A.

The diameters of the objects were measured with the screws of the double micrometer M in the two positions of the plate at 0° and at 180° . The values reduced to seconds of arc are inscribed in the columns 2a, 2b.

The degree of luminosity, or the density of the images measured, was obtained by comparison with the Isaac Roberts' scale for stellar magnitudes, which is a photographic plate containing twenty-five little squares corresponding to diverse time exposures.

Squares: 1, 2, 3, 4, 5, are opaque, very dense, decreasing in opacity.
 6, 7, 8, 9, 10, are dense, decreasing in opacity.
 11, 12, 13, 14, 15, are pretty dense.
 16, 17, 18, 19, 20, are translucent, increasing in transparency.
 21, 22, 23, 24, 25, are transparent, increasing in transparency.

The numbers inscribed in column headed "Intensities" are but approximate values of the intensity of the images measured, the estimation being very delicate, as the densities of the little squares blend almost imperceptibly. The measured rectangular co-ordinates, inscribed in the columns x and y , are referred to the centre of the plate and are expressed in minutes and fractions of a minute of arc; the values of a revolution of the screws H and G were obtained by comparing measures of distances, made with the screws H and G, of certain stars in the group of the Pleiades with the corresponding values obtained by Prof. Elkin.*

The values inscribed in the columns Δx , Δy , are the measures expressed in minutes and fractions of a minute of arc and referred to the nucleus of the nebula; the values under the title "Star A" are referred to Star A.

The columns R.A., Decl., contain the concluded positions for 1900 of the objects measured. The seconds and fractions of a second only are given for the positions derived from plate I.R. 1727 and for the adopted places of the Standard Stars from which the constants of the plates were derived; the places adopted were those of the Catalogue of the Astronomische Gesellschaft, no

* "Researches with the Heliometer, Determination of the Relative Positions of the Principal Stars in the group of the Pleiades."

positions from the Astrophotographic Catalogue being available for the region of the nebula.

The four constants and the concluded centre for I.R. 3197 and I.R. 1727 are inscribed in the Table, which contain also remarks concerning the definition of the measured images. The differences in R.A. and Decl. between I.R. 3197 and I.R. 1727 are of the same order, whether the faint condensations involved in the nebula or the faint stars surrounding the nebula are considered; the measures here given do not reveal any change that may have taken place in the nebula during the interval of ten years that separates the two exposures.

It may be of interest to compare the results obtained from the Isaac Roberts' plates with the observations of the nebula H.I.159 made at Birr Castle and published in Lord Rosse's *Observations of Nebulæ and Clusters*, 1848-1878, p. 13. The observations made in 1848, 1849, 1850, 1855, and 1866 read as follows:

G.C. 158, *h*71, H.I.159.

1848. Dec. 18. R, *b*MN F* about one diameter off to *n*;
three to the south, of which one is *p*B and the two
others *v*F.

The stars referred to by Lord Rosse are the stars marked in my diagram and in the table of results annexed, by 19, star to the *np*, and stars 7, 8 and 9 to the *sf*, the brightest being star 7.

1849. Nov. 13. About 80" diameter with sharp nucleus, 2 *st*
almost certain, a third very doubtful. Inequality in
the light suspected?

The diameters of the nebula, as measured on the I.R. plates, are from *s* to *n*, including the nebulous projection or extremity of the spiral,

57" and from *p* to *f* 49" (plate I.R. 1727)
60" ,, ,, 49" (plate I.R. 3197)

These values refer to the denser parts of the nebula, and do not include the extremely faint nebulous outlines, which seem to contain nebulous knots considerably fainter than those measured and marked on the diagram. The faint nebulous projection preceding condensation 2 appears to be formed by two of these extremely faint nebulous knots of the outer nebulous region of H.I.159.

The diameters of the central nucleus are:

13", 11" (3197)
13", 10" (1727)

The stars and condensations on the negatives are slightly elongated.

Professor E. Becker (pages 53 and 91 of the *Annalen der Kaiserlichen Universitäts-Sternwarte in Strassburg*) gives the

values $1'$ (July 30, 1876) and $40''$ (Nov. 12, measured by moonlight).

Sir John Herschel finds $20''$ (Smyth and Chambers's *Cycle of Celestial Objects*).

"The two stars almost certain" referred to by Lord Rosse appear to be the condensations 7 and 8 of my diagram; they stand out more conspicuously than the other nebulous knots, because they appear upon a lighter background, and also because they seem to be free from dense nebulosity on their f margin. North of condensation 8 is a group of nebulous knots of which the brightest is condensation 9 of my diagram.

"The third very doubtful star" mentioned by Lord Rosse appears to be condensation 5 of my list and diagram. It appears less distinct than 7 and 8, because it is involved in nebulosity of a denser nature, and completely immersed in the nebulosity.

"The suspected inequality in the light" is confirmed by the Isaac Roberts' photographs; the inter-spiral space in the nf region of the nebula is less dense than the nebulosity forming the main part of the nebula.

1850. Sept. 12. Marked by h planetary, but has no decided nucleus.

1850. Oct. 1. Seems to have a Nucl.

1855. Oct. 16, pB , R , N , certainly patchy, but I can distinguish no stars in it. It is a right-handed spiral?? I suspect a $F^* cl, f$ the neb.

We here see that, in 1855, Lord Rosse was in possession of the correct interpretation of the nebula, which is a spiral nebula, but a left-hand spiral. "The faint star close following the nebula" is a nebulous condensation, $C7$ mentioned above.

1866. Sept. 14, B , pL , mbM . A^* in pos. about 350° dist. $145''$ (VII. obs.).

A doubt as to the spirality of the nebula seems to take hold of the observer. Were the observations of 16th Oct. 1855 and of 14th Sept. 1866 made by the same observer?

"The star about in pos. 350° and dist. $145''$ " is star A of my diagram: $B.D. +46^\circ 183$. Lord Rosse's position reduced to 1900 is: Pos. Angle: $350^\circ 3'3$ dist. $145''$, or

$$\begin{aligned}\Delta R.A. &= 2^s 44 \\ \Delta Decl. &= 2' 22'' \cdot 8\end{aligned}$$

The positions measured on

$$\begin{array}{rcll} I.R. 3197 & \text{give } \Delta R.A. = & 1^s \cdot 19 & \Delta Decl. 2' 37'' \cdot 1 \\ 1727 & ,, & ,, & 1 \cdot 28 \quad . \quad 2' 38'' \cdot 6 \end{array}$$

The photographs show that within a radius of $6'$ of are there are but very faint stars (star A being excepted) in the p and np

outer regions of the nebula H.I.159, whereas the stars situated in the other exterior regions are much brighter. Between stars 1 and 19 (see diagram) is a faint nebulous patch visible both on I.R. 3197 and 1727, and measured on both negatives. Is it of a nebulous nature, or is it formed by faint stars? Two other nebulous patches of similar aspect were measured on plate I.R. 3197. These nebulous masses are in a region void of bright stars, and where the faint stars are few in number.

It would be interesting to know whether these objects are really of a nebulous structure, and if so, whether they are derived from the nebula H.I.159.

More powerful instruments than the one used at Starfield by the late Doctor Isaac Roberts and his assistant, Mr. W. S. Franks, will throw new light upon this question.

With my gratitude to the memory of Isaac Roberts, I here wish to express my thanks to M. Baillaud, Directeur de l'Observatoire National, Paris; to M. Lagarde, Chef du Bureau des Calculs, Observatoire de Paris; to the Secretary of the Royal Astronomical Society, Mr. A. R. Hinks, for valuable advice and criticism; and to M. Pourteau, of the staff of the "Bureau des Calculs," and to my assistant of By-Thomery, Madeleine Bougréau, who have assisted me in my task.

Rectangular Co-ordinates Measured on the Plates.					Concluded Positions for 1900.0 of the Standard Stars.									
I.R. * 3197.		I.R. * 1727.			I.R. * 3197.				I.R. * 1727.					
x.	y.	x.	y.		R.A.		Decl.		R.A.		Decl.			
					h	m	s	°	'	"	s	'	"	
0	0'000	0'000	0'000	0'000										
B	- 26'968	+ 14'366	- 38'495	+ 16'448	0	44	1'04	+ 47	13	9'3	1'03			9'
G	- 5'166	+ 31'745	- 16'686	+ 33'821	46	8	7'1		30	44'8	8'68			44"
A	- 2'580	+ 3'995	- 14'087	+ 6'075	46	25	0'1		3	1'6	25'03			1"
E	+ 6'206	+ 14'802	- 5'296	+ 16'891	47	16	3'7	47	13	52'0	16'40			52"
D	+ 6'373	- 25'757	- 5'127	- 23'665	47	18	1'2	46	33	16'7	18'13			19"
F	+ 34'776	- 7'764	+ 23'278	- 5'655	0	50	03'91	46	51	16'5	3'85			17"
C	+ 1'985	- 12'062	- 9'516	- 9'979	0	46	52'24	46	47	00'0	52'27	46	59	"
Neb.	- 2'393	+ 1'376	- 13'883	+ 3'431	0	46	26'20	+ 47	0	24'5	26'31			22"

Condensations involved in *H.I.159 Cassiopeiae* and

Measured Co-ordinates.				Concluded Places for 1900.0.									
I.R. * 3197.		I.R. * 1727.		I.R. * 3197.				I.R. * 1727.					
$\Delta x.$	$\Delta y.$	$\Delta x.$	$\Delta y.$	R.A.			Decl.			R.A.	Decl.		
				h	m	s	°	'	''	s	''		
c''_3	- 0'319	- 0'125		0	46	24'33	+ 47	0	16'9				
c_3	- 0'247	- 0'052				24'75			21'3				
c'_3	- 0'185	- 0'018				25'12			23'4				
[c_3]		- 0'335	- 0'029			[24'73]			[20'5]	24'35		20'8	
c_2	- 0'245	+ 0'166				24'75			34'4				
c'_2	- 0'156	+ 0'196				25'28			36'2				
[c_2]		- 0'204	+ 0'191			[25'01]			[35'3]	25'11		34'0	
c_4	- 0'200	- 0'340				25'04			4'0				
c_1	- 0'082	+ 0'148				25'71			33'4				
c'_1	+ 0'060	+ 0'277				26'55			41'1				
[c_1]		- 0'024	+ 0'211			[26'13]			[37'2]	26'17		35'2	
c_5	+ 0'128	- 0'284	+ 0'086	- 0'243		26'96			7'5	26'82		8'1	
c_9	+ 0'219	+ 0'394	+ 0'216	+ 0'403		27'47			48'2	27'56		46'5	
c_8	+ 0'236	+ 0'223	+ 0'234	+ 0'254		27'58			37'9	27'68		37'9	
c_6	+ 0'253	- 0'197	+ 0'239	- 0'124		27'69			12'7	27'72		15'1	
c_7	+ 0'342	- 0'011	+ 0'374	+ 0'020	0	46	28'20	+ 47	0	24'0	28'50	23'1	
p_1	- 1'455	+ 0'381	- 1'452	+ 0'405	0	46	17'65	+ 47	0	46'9	17'78	46'1	
p_2	- 4'609	+ 0'019			0	45	59'18		47	0	23'8		
p_3	- 6'069	+ 0'365			0	45	50'59	+ 47	0	43'9			

Adopted Places for 1000'0 of the Standard Stars
from which the Constants were derived.

Diameters of Star Images.

	R.A.	Decl.	B.D.	Mag. B.D.	I.R. * 3197.		I.R. * 1727.	
					2a.	2b.	2a.	2b.
	s	"	°		"	"	"	"
B	1°11	10°5	176+46	6°5 G	43	34	37	32
G	8°73	43°2	226+47	7°2 A	33	30	28	27
A	24°89	1°6	183+46	8°5 A	31	26		
E	16°35	52°8	187+46	8°8 A	25	21	20	19
D	18°15	18°5	189+46	7°9 A	33	32	26	28
F	3°94	17°3	201+46	8°0 A	36	29	26	23
C			186+46	9°1 B	27	23	21	19
Neb.			H.I. 159		13	11	13	10

Measured with respect to the Nucleus of the Nebula.

I.R. * 3197.	I.R. * 1727.	Diameters of Images.		Definition and description.
		I.R. * 3197.	I.R. * 1727.	
Intensities.		2a.	2b.	
		"	"	"
c''_3		7	12	Elongated n to s .
c_3	12 16	14	12	Involved in dense nebulosity. Multiple nuclei. F rays directed toward n .
c'_3		13	8	Dense nucleus; close to N.
$[c_3]$			10 9	Mean value of c''_3 , c_3 , c'_3 . c_3 on I.R. * 1727 is involved in dense nebulosity.
c_2	13 15	8	6	
c'_2		5	4	Perhaps part of c_2 .
$[c_2]$			11 6	Mean value of c_2 c'_2 . On I.R. * 1727 E, near margin of nebula; F nebs arc in p region.
c_4	10°15	9	10	Near margin of neb; dense in n reg; fainter in s region.
c_1	13 18	8	6	e F; involved in dense neb'y.
c'_1	13	9	7	Involved in neb'y; nr border of spiral, n region.
$[c_1]$			9 7	Mean value of c_1 c'_1 . On I.R. * 1727; e F involved in inner spiral.
c_5	8 14	8	7	Dense E.N. involved in neb'y, nr margin of nebula; 1727: multiple nuclei.
c_9	13 15	10	10	6 7 Multiple nuclei, 1727: e F in outer spiral.
c_8	12 13	8	7	8 7 F. F in outer spiral.
c_6	13 15	10	9	8 10 Near margin of nebula. Very diffuse; difficult.
c_7	9 12	13	10	9 4 Multiple nuclei due to exposure. 1727: nr margin of nebula.
p_1	19 19.	15	10	14 11 Nebulous patch—perhaps v F. * or group of stars.
p_2	19	7	7	" " " "
p_3	17	13	11	" " " "

The Constants and Concluded Centre of the Plates.

I.R. * 3197.

I.R. * 1727.

$$t_x = -0.00009$$

$$t_y = -0.00045$$

$$t_x = -0.00061$$

$$t_y = -0.00019$$

$$i_x = -0.00519$$

$$i_y = -0.00508$$

$$i_x = -0.00148$$

$$i_y = -0.00117$$

$$\alpha_0 = 0^h 46^m 40^s.28 \quad \delta_0 = +46^\circ 59' 2''.8 \quad \alpha_0 = 0^h 47^m 47^s.7 \quad \delta_0 = +46^\circ 56' 59''.$$

Faint Stars in the Vicinity of the Nebula measured with respect to the Nucleus of the Nebula
Concluded Places for 1900.0.

I.R. * 3197.		I.R. * 1727.		I.R. 3197.		I.R. 1727.		I.R. * 3197.	I.R. * 1727.
$\Delta x.$	$\Delta y.$	$\Delta x.$	$\Delta y.$	R.A.	Decl.	R.A.	Decl.	Intensity	
2. -2'187	-1'233	-2'181	-1'208	^h 0 ^m 46 ^s 13.42	+46° 59' 9".8	^s 13.57	9".4	14	12.
1 -1'479	-0'735	-1'468	-0'727	17.55	46 59 39.9	17.73	38.5	13	11
3 -1'268	-2'319	-1'297	-2'303	18.85	46 58 5.0	18.79	4.0	16	16
5 -1'074	-3'440			20.03	46 56 57.9			15	
19 -0'712	+1'396	-0'707	+1'422	21.97	47 1 48.0	22.12	47.7	9.12	10
4 -0'709	-2'383	-0'708	-2'362	22.13	46 58 1.4	22.25	0.7	17	19
6 -0'064	-3'594			25.95	46 56 49.0			3	
18 +0'389	+1'574	+0'403	+1'627	28.43	47 1 59.1	28.62	2 0.4	14	17
7 +0'537	-2'018	+0'549	-1'992	29.41	46 58 23.7	29.60	23.3	6	4
8 +0'763	-1'611	+0'750	-1'587	30.72	46 58 48.2	30.76	47.7	10.17	8
15 +0'966	+0'973			31.83	47 1 23.2			17	
9 +0'969	-1'478	+0'949	-1'460	31.93	46 58 56.2	31.92	55.3	9	9
16 +1'065	+1'331			32.40	47 1 44.7			19	
14 +1'139	+0'901			32.85	47 1 19.0			19	
10 +1'288	-0'595	+1'301	-0'559	33.77	46 59 49.3	33.96	49.5	15	16
17 +1'339	+2'166			33.99	47 2 34.9			13	
13 +1'398	+0'361	+1'423	+0'442	34.38	47 0 46.6	34.65	49.6	7	5
11 +1'704	-0'322	+1'688	-0'295	36.21	47 0 5.8	36.21	5.5	14	15
12 +2'306	-0'013			39.73	47 0 24.5			7	

Same Stars measured with respect to Star A. B.D. +46° 183.

I.R. * 3197.		I.R. * 1727.		Concluded Places for 1900.0.		Diameters.			
$\Delta x.$	$\Delta y.$	$\Delta x.$	$\Delta y.$	R.A.	Decl.	R.A.	Decl.	3197.	1727.
2 -2'014	-3'857	-2'002	-3'857	^s 13.35	' 9".6	^s 13.43	' 9".1	" 8"	" 7"
1 -1'298	-3'351	-1'277	-3'364	17.51	40.2	17.66	39.0	9	6
3 -1'077	-4'938	-1'113	-4'952	18.87	5.1	18.68	3.8	8	5
5 -0'882	-6'052			20.05	58.4			9	6
19 -0'521	-1'212	-0'495	-1'219	22.00	48.8	22.17	47.9	9	6
4 -0'494	-5'013	-0'514	-5'010	22.29	0.8	22.19	0.5	7	4
6 +0'147	-6'216			26.09	48.9			15	11
18 +0'595	-1'023	+0'630	-1'022	28.53	2 0.4	28.76	2 0.1	8	7
7 +0'730	-4'642	+0'745	-4'641	29.45	23.5	29.55	23.0	11	7
8 +0'941	-4'230	+0'949	-4'237	30.67	48.2	30.73	47.2	8	6
15 +1'156	-1'643			31.85	23.5			5	6
9 +1'140	-4'107	+1'147	-4'107	31.83	55.7	31.89	55.1	8	6
16 +1'257	-1'284			32.43	45.0			11	7
14 +1'346	-1'712			32.97	19.4			7	8
10 +1'483	-3'227	+1'500	-3'219	33.82	48.6	33.93	48.6	10	6
17 +1'537	-0'450			34.04	35.1			9	5
13 +1'597	-2'240	+1'639	-2'250	34.46	47.8	34.71	46.7	11	7
11 +1'897	-2'937	+1'898	-2'934	36.24	6.1	36.25	5.8	11	7
12 +2'493	-2'620			39.73	25.3			9	7

Maison de Rosa Bonheur, By-Thomery
1912 February 3.

The Long-Period Variable RT Cygni (Ch 7085) in 1911.

By A. N. Brown, M.A. (Plate 4.)

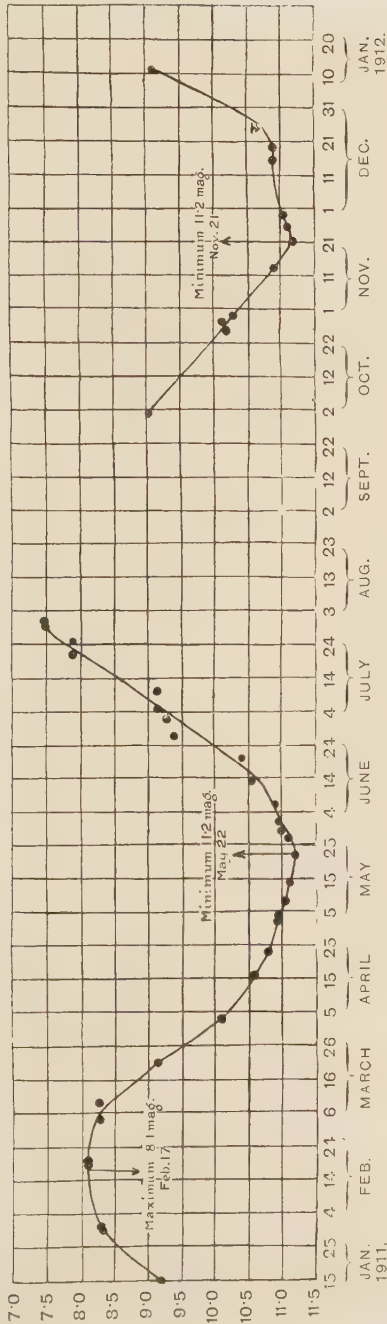
A continuation of my observations of this Variable, the last series of which appeared in *Monthly Notices*, vol. lxxi. No. 5, is given below. The light curve drawn from them shows one maximum on February 17 and two minima on May 22 and November 21 respectively, all of which seem clearly defined. Another maximum undoubtedly occurred during the first half of August, but, as during this and the following month I was making a rapid journey across Canada, I was unfortunately unable to secure any observations between July 31 and October 1. I find, however, that the average time of ascent from minimum to maximum from 1908 April 2 to the date in question has been approximately 82 days. In all probability therefore the maximum in the summer of 1911 took place about August 12; and the observations made during July and October support this conjecture.

The maximum of 1911 February 17 is the faintest of any which I have observed, the average brightness of the Variable at the six preceding maxima according to my results having been 7.3 mag.; and it is possibly worth noting that this faint maximum immediately follows an unusually slow fall to minimum light in 1910 November. The average time occupied by each of the four descents from maximum to minimum preceding this was 106 days. On this occasion the fall took 130 days. It will be interesting to notice in the future whether these conditions are repeated.

The observations, which number 42, are arranged as follows:—In the first column the date is given; in the second the Julian date, omitting the constant number 2,410,000; in the third the instrument used in making the observation. In this column the letters T, t, and p respectively indicate refractors of 5-in., 3-in., and 2-in. aperture, each letter being followed by the magnifying power employed. Most of the comparisons given in the fourth column were made by the “fractional” method, but each is here expressed in steps, the value of a step in every case being .1 magnitude. All the comparison stars used are numbered as in Hagen’s Atlas, the negative sign after the comparison star implying that the Variable was fainter, the positive sign that it was brighter, than the star in question. A semicolon has always been inserted between each separate estimate. Thus on May 4 one fractional observation is shown; on May 8 an estimate was made by the step method; on October 1 the observation consisted of two fractional estimates. As regards the magnitude of each comparison star, the Harvard value, where this was available, has been adopted; when the H.C.O. does not assign a value to one of Hagen’s stars used, Hagen’s own value, as given in the *Atlas Stell. Var.*, has been taken. In the fifth column the magnitude deduced from the comparisons is given, a mean being taken where more than one estimate has been made. As all these observations were considered reliable, the

column showing the "class" included in previous series has been omitted, but no definite magnitude was assigned to the Variable on December 25. This is shown in the curve by a small arrow, which is intended to indicate that the light was certainly fainter than 10.6 magnitude on that evening.

<i>The Observations.</i>				
Date.	Julian date.	Instr.	Comparisons.	Ded. mag.
1911.				
Jan. 15	9052	t. 60	16-4.5, 19+1	9.2
30	9067	T. 50	7-3, 14+3	8.3
31	9068	"	7-3, 13+2	8.3
Feb. 19	9087	t. 60	7-1, 10+2.5	8.1
20	9088	"	7-1, 10+3	8.1
Mar. 4	9100	T. 50	7-3, 10+1	8.3
9	9105	"	7-3, 10+1	8.3
21	9117	t. 60	16-4.5, 19+1.5	9.2
Apr. 3	9130	T. 50	30-1, 41+3	10.1
16	9143	t. 120	=43	10.6
23	9150	"	43-2.5, 44+2	10.8
May 2	9159	T. 120	43-3.5, 44+1	10.9
4	9161	T. 167	43-3.5, 44+0.5	11.0
8	9165	T. 200	44-0.5	11.1
14	9171	T. 167	44-1	11.1
22	9179	T. 200	44-2	11.2
27	9184	"	44-1	11.1
29	9186	"	=44	11.0
June 1	9189	"	44+0.5	11.0
6	9194	"	43-2.5, 44+1	10.9
13	9201	"	43+0.5	10.6
20	9208	"	41-0.5, 43+2	10.4
27	9215	T. 50	19-1, 22+1	9.4
July 2	9220	"	=19	9.3
5	9223	"	16-4.5, 19+1.5	9.2
10	9228	"	16-4.5, 19+1.5	9.2
21	9239	"	5-1.5, 7+1	7.9
25	9243	"	5-1, 7+1	7.9
30	9248	t. 30	3-8.5, 4+1	7.5
31	9249	"	3-8, 4+1.5	7.5
Oct. 1	9311	T. 50	15-4, 20+1; 16-4, 19+2	9.1
Oct. 25	9335	T. 120	30-2, 41+2	10.2
28	9338	"	30-1.5, 41+3	10.1
30	9340	"	41+1	10.3
Nov. 13	9354	T. 200	43-3, 44+1	10.9
21	9362	T. 150	44-2	11.2
25	9366	"	44-1	11.1
29	9370	T. 200	44-0.5	11.1
Dec. 15	9386	"	44+1	10.9
19	9390	T. 167	43-3, 44+1	10.9
25	9396	p. 26	<43. Var. and 44 glimpsed	<10.6
1912.				
Jan. 11	9413	t. 60	15-3, 20+2; 16-4.5, 19+1	9.1



LIGHT CURVE OF RT CYGNI, 1911.—A. N. BROWN.

The Comparison Stars used.

Hagen's number.	Magnitude.		Hagen's number.	Magnitude.	
	H.P.	Hagen.		H.P.	Hagen.
3	6.67		16	8.71	
4	7.64		19		9.3
5		7.8	20		9.3
7		8.0	22		9.5
10	8.39		30		10.0
13	8.54		41		10.4
14	8.67		43		10.6
15		8.8	44		11.0

Ludgrove, New Barnet, Herts :
1912, Jan. 15.

A New Variable Star. By the Rev. T. E. Espin, M.A.

On Dec. 18, 1909, while observing Σ 446, I noticed a red star preceding which was not in the B.D. The spectrum was evidently banded and apparently III.!! 1910 Jan. 10 the spectrum was again observed, and was entered as Bands III.? On 1911 Jan. 25 the spectrum was entered as IV.? The following observations clearly show variation, though they are not sufficiently numerous to determine the amount or period:—

1909 Dec. 18	About 9.2
20	= B.D. + 51° 770 . . 9.0
24	9.1
31	9.1
1910 Jan. 10	8.9 Brightest star in field
Feb. 18	9.3
1911 Jan. 25	9.7
27	10.2
1912 Feb. 10	10.2
23	10.3
Mar. 12	10.5

The position of the star as determined from Σ 446 is

R.A. 3^h 39^m 45^s Decl. + 52° 21'.8 (1900).

*The Effect of Atmospheric Dispersion on the Greenwich
Photographs of Eros.*

(Communicated by the Astronomer Royal.)

As the effect of atmospheric dispersion introduces some uncertainty into the determination of the solar parallax from photographic observations of minor planets, it seems worth while to indicate the amount by which the parallax deduced from the Greenwich Observations would be altered on the assumption that the mean wave-length of the light giving the photographic image of the planet and of a comparison star differs by a definite quantity, say 10 tenth-metres.

Kayser and Runge (*Handbuch der Spectroscopie*, Band iii. S. 513) give the following formula for the refractive index of air at 0°C. and 760^{mm}:

$$10^7(n-1) = 2878\cdot7 + 13\cdot16 \lambda^{-2} + 0\cdot316 \lambda^{-4},$$

where λ is measured in thousandths of a millimetre (μ).

This formula gives

$$\begin{array}{ll} \text{D} & \lambda = 5900 : 10^7(n-1) = 2919\cdot3 \\ \text{E} & \lambda = 5270 : 10^7(n-1) = 2930\cdot2 \\ \text{G} & \lambda = 4340 : 10^7(n-1) = 2957\cdot5 \\ \text{and at G} & 10^7 \delta n = -0\cdot0401 \delta \lambda \end{array}$$

where λ and $\delta \lambda$ are expressed in tenth-metres.

The difference of refraction is given by

$$\begin{aligned} \delta r &= \delta n \tan z \\ &= 10^{-7} \times 0\cdot0401 \delta \lambda \tan z \\ &= 0\cdot00083 \delta \lambda \cdot \tan z. \end{aligned}$$

Thus for $\delta \lambda = 10$ tenth-metres

$$\delta r = 0\cdot008 \text{ at } 45^\circ \text{ Z.D. and } 0\cdot014 \text{ at } 60^\circ \text{ Z.D.}$$

and quantities of this order would have a sensible effect on the parallax.

Let P be the parallax of Eros in R.A. and Δ the planet's distance.

$$\text{Then} \quad P = 8''\cdot8 \frac{1}{\Delta} \sin z \cos S.$$

The displacement due to atmospheric dispersion is

$$\begin{aligned} & \delta n \tan z \cos S \\ &= \delta n P \sec z \cdot \frac{\Delta}{8''\cdot8}. \end{aligned}$$

Thus the correction to the solar parallax deduced from two observations of Eros at zenith distances z_1 and z_2 and with parallax factors in R.A. P_1 and P_2 respectively is

$$\delta\pi = \delta n \frac{P_1 \sec z_1 - P_2 \sec z_2}{P_1 - P_2} \cdot \Delta.$$

Applying this formula to table 25, p. lxii., in the *Greenwich Observations of Eros*, the following table is obtained:—

Date.	Correction to Solar Parallax, 8''800.	Weight.
1900 Oct. 14-15	— '045 + '90 δn	8
20	+ '117 + '58	13
21	+ '007 + '67	70
26	+ '006 + '61	150
27	+ '038 + '59	139
28-29	— '009 + '60	97
Nov. 8-9	— '012 + '43	36
9-10	+ '012 + '52	109
10-11	+ '012 + '55	88
13	— '011 + '55	196
14-15	— '067 + '42	108
22-23	+ '009 + '46	174
27	— '005 + '39	50
Dec. 6-7	+ '054 + '54	58
9-10	+ '011 + '38	47
13	— '044 + '49	88
15-16	— '018 + '46	203
19	+ '003 + '45	156
21	+ '033 + '43	109
28-29	+ '033 + '63	22
1901 Jan. 5	'000 + '69	35
8-9	— '082 + '60	82
13-14	+ '030 + '58	73
15	+ '034 + '56	100
Mean	''000 + '523 δn	2211

The probable error of this result is $\pm$ ''0044.

Since $\delta n = ''00083 \delta\lambda$ for a difference of wave-length $\delta\lambda$, the correction to the solar parallax is ''00043 $\delta\lambda$: thus if $\delta\lambda = 10$ tenths-metres, the correction amounts to ''0043. The observations given

above have not sufficient weight for any result to be obtained by comparing the results given by large and small coefficients of δn .

Attention may be drawn to the method employed by M. Prosper Henry to determine the value of $\delta\lambda$, given in Paris Circular No. 8. By means of a grating of parallel bands 5^{mm} wide with spaces of 5^{mm} placed in front of the object glass of an astrographic telescope, he obtained diffraction images of the planet and comparison stars in which the dispersion was small and comparable with that given by the atmosphere. By measurement of the distance between the two first diffracted images on opposite sides of the direct image, he obtained a measure of the mean wave-length of the light which went to form these images. From two sets of observations he obtained a mean difference of 20 tenth-metres between the light of the planet and surrounding comparison stars, the planet's light having smaller wave-length, and regarded this as sufficiently near. In view of the accuracy with which the solar parallax has, apart from the effect of atmospheric dispersion, been obtained, it would seem desirable to repeat these experiments at the next opposition of Eros.

*Occultations of B.A.C. 1189 and of Mars, observed at
Cambridge Observatory, 1912 Jan. 28.*

(Communicated by the Director.)

The observations were made with the Northumberland Equatorial by Mr. Hinks. His clock correction depends on the mean of eye and ear and of chronograph transits taken between the occultations.

		^h ^m ^s
*B.A.C. 1189	Disappearance	G.M.T. 13 44 28.5
Mars	Occultation already begun	14 32 21.9
	Estimated bisection	24.4
	Disappearance	31.9

The altitude of the planet was only about 7° , and the image was strongly fringed with colour. Definition very poor.

Cambridge Observatory:
1912 Jan. 31.

*Position of Comet Beljowsky (1911 g), observed at Cambridge Observatory.**(Communicated by the Director.)*

The observations were made by Mr. Hinks with the 5-inch Micrometer Theodolite from a point in the observatory grounds whose coordinates are lat. $52^{\circ} 12' 48''$, long. $0^{\text{m}} 22^{\text{s}}.8$ E. Each is the mean of a pair of pointings with Face L. and Face R. The times were taken with a half chronometer watch. The zero of azimuth was obtained from a pair of observations of β Leonis immediately following the comet observations. The deduced places are corrected for refraction, but not for parallax.

	G.M.T.			Obs. Altitude.			Azimuth (from S. by W.)			App. R.A.			App. Decl.		
1911.	h	m	s	°	'	"	°	'	"	h	m	s	°	'	"
Oct. 4	16	34	59.8	5	42	40	257	52	17	11	50	48.0	+	11	55 53
		43	27.3	6	58	43	259	30	34	11	50	50.9		11	55 57

The observations were made with considerable difficulty, on a hazy sky with streaks of cloud, and interruption by dawn.

Cambridge Observatory:
1912 Feb. 3.

The Real Paths of 429 Fireballs and Shooting Stars observed in the British Isles during the 15 Years 1897 to 1911 inclusive.
By W. F. Denning.

(Continued from Monthly Notices, January 1897.)

The following are the heights, radiant points, etc., of such meteors as have been observed sufficiently well in this country* for their real paths to be investigated. The period extends from January 1897 to December 1911. A similar table of results, including 107 meteors, for the 10 years 1886 to 1896, was published in the *Monthly Notices* for January 1897. A number of the details presented in the present paper have been printed before in a fragmentary form in the *Memoirs* and *Journal of the British Astronomical Association*, in *The Observatory*, *Astronomische Nachrichten*, etc. It is thought, however, that by collecting them all together and arranging them in a convenient form for reference, their utility to future students of the subject might be enhanced. The meteors have therefore been summarised according to the dates of their apparition, as it is thought the plan may enable ready comparisons to be made with the materials in past and future lists. There is no question that certain special epochs are very productive of fireballs, and it is desirable that the dates should be definitely learnt, and the radiant points of the more

* Three meteors observed abroad have been included, viz., 1909 April 20 Lyrid, 1908 May 27 Aurigid, and 1908 Oct. 23 Arietid.

active streams determined. This has already been effected in several cases, but the data are not sufficiently full, and our knowledge is still very far from approaching completion.

In some instances fireballs of the largest type appear to be intermingled with the faintest visible shooting stars. In others, large meteors seem to be isolated and quite unattended by small meteors. There are also systems which apparently consist mainly of large fireballs. Again, on various dates well known for the frequency of fireballs no special showers can be recognised, the flights exhibit a variety in their directions sufficiently proving the objects to belong to a number of different systems.

A very considerable number of the objects included in the list were seen and recorded by one or more experienced observers, and the deductions from such materials may be depended upon as being as accurate as the nature of the work allows. The observations are only eye estimates of swiftly moving objects, but it is astonishing how nearly correct these can be made after the necessary practice.

England, compared with other countries, affords unusual facilities for securing multiple observations of the same meteors. Her small geographical area, and the relative proximity of populous towns, render it difficult for any bright fireball to elude notice by several qualified persons when the sky is clear enough for the purpose. An object appearing over central England admits of observation over nearly the whole of the United Kingdom if the altitude and brightness of the meteor are pretty considerable. This is an advantage which has made itself appreciable in the large number of interesting and conspicuous meteors which have been sufficiently well observed from this country for their real paths to be computed.

Radiant points ascertained from double or multiple and accurate observations of a meteor are valuable because they are unquestionable. But radiant points derived by an observer from his own observations singly must often be somewhat uncertain, as he can only presume where the points lie from the directions and apparent aspects of the objects recorded. And, even after many years of experience, errors in attributing the radiants are unavoidable, and are frequently made though the observer is careful to consider every detail helping him to form a correct conclusion. In ordinary cases no radiant point should be accepted which is not based on 5 or 6 paths at least, though there are occasions when, a radiant being near the horizon, 3 or 4 meteors may be sufficient for safe results, and especially when the observer has had an extensive practice in the allocation of meteors to their radiants. A great deal necessarily depends upon experience and individual aptitude; there are some observers who are far more accurate than others, both in registering the apparent flights of meteors and in apportioning their radiants.

I have not attempted to analyse the results in the list or to

make any statistical deductions.* As the results are still accumulating, such investigations are perhaps better left until more materials are available. And it is to be hoped that in the immediate future a more general effort will be made, both by astronomical and meteorological observers, to fully record the bright meteors which they incidentally witness. Fireballs are more abundant than is commonly supposed, and, when suitably and exactly recorded, supply us with important evidence on various points connected with this branch of astronomy. Yet great numbers of the most brilliant and significant of these objects, from lack of suitable description, have been lost to science and relegated to oblivion. In instances when it is fair to conclude that a large meteor had some thousands of spectators, only two or three very imperfect accounts have been furnished of its flight. Fortunately, the present-day observer is being led to recognise the importance of such phenomena, and the utility of recording details of their appearance.

Several meteors in the table yield heights of more than 100 miles at the beginning of the flights, and I think such extreme elevations are to be regarded with distrust. A small error in putting a meteor a few degrees too far back in the line of its visible course will sometimes occasion an excessive height of 10 or 15 miles. In exceptional cases, where the swift streaking meteors are concerned, the height may possibly be beyond 100 miles, but there are very few well-supported instances. I believe that the Perseids and Leonids usually become luminous at about 80 miles and are spent at 50 miles. It is a rare thing for one of them to penetrate below 40 miles. The slow meteors fall to much lower elevations, and many of them are not more than about 30 miles high at extinction.

Bristol :
1912 February 14.

The abbreviations are :—

T.H.A.,	T. H. Astbury.	W.F.D.,	W. F. Denning.
W.E.B.,	W. E. Besley.	A.S.H.,	Prof. A. S. Herschel.
T.W.B.,	T. W. Backhouse.	S.J.J.,	Rev. S. J. Johnson.
J.H.B.,	J. H. Bridger.	A.K.,	A. King.
C.L.B.,	C. L. Brook.	T.E.R.P.,	Rev. T. E. R. Phillips.
H.C.,	H. Corder.	Mr. and Mrs. W.,	Mr. S. A. Wilson and Mrs. Fiammetta Wilson.

Cols. 1 and 2 give the date.

3 the hours and minutes of G.M.T.

4 the estimated magnitude.

5 the height in miles of the point of appearance.

6 the height in miles of the point of disappearance.

7 the length in miles of the observed path.

8 the velocity of flight in miles per second.

9 and 10 the R.A. and Decl. of the radiant point in degrees.

* I gave some particulars respecting the average heights of fireballs in: *Knowledge* for 1901 December.

Date.	G.M.T.	Mag.	App.	Dis.	Path.	Vel.	Radiant.
	h m		miles.	miles.	miles.	m.p.s.	° °
1900 Jan. 2	6 41	3	69	60	35	24	229 + 52
1900 Jan. 2	11 58½	> 1	57	40	44	22	228 + 53
1903 Jan. 3	7 25	2	60	47	52	16	228 + 52
1903 Jan. 3	9 5	2	71	49	88	29	229 + 50
1903 Jan. 3	12 5	1 > 1	65	49	41	26	228 + 53
1903 Jan. 3	12 59½	1 < 2	67	54	30	20	227 + 50
1900 Jan. 9	2 55	F	59	23	174	...	280 - 12
1904 Jan. 9	8 28	2 > 2	60	41	27	6	41 + 5
1904 Jan. 10	8 33	2	67	31	43	21	43 + 22
1900 Jan. 10	9 30	½ = 1	63	31	62	18	116 + 2
1909 Jan. 11	8 11	> 2	58	29	52	13	332 + 36
1903 Jan. 13	6 15	2	57	54	200	34	218 + 39
1901 Jan. 13	10 35	F	61	52	54	8	149 - 12
1903 Jan. 14	7 7	2 < 2	57	54	53	21	120 - 3
1905 Jan. 14	10 16	2	60	29	55	15	119 + 3
1904 Jan. 15	9 22	2	63	27	36	12	61 + 41
1898 Jan. 21	5 32	F	82	25	235	34	130 + 30
1903 Jan. 25	7 59	2	95	16	125	15	99 + 13
1900 Jan. 25	11 10	3 1	65	44	26	30	45 + 62
1909 Jan. 25	13 25	2	55	42	108	16	40 + 18
1911 Jan. 25	7 7	2	83	46	80	21	330 + 58
1900 Jan. 27	11 10	1½ 1	57	45	28	14	98 - 10
1905 Jan. 27	11 58	½ 1	66	33	53	7	118 0
1906 Jan. 27	8 33	= 1	59	45	42	24	214 + 53
1901 Jan. 28	9 18	2 < 2	76	16	61	18	106 + 52
1903 Jan. 28	11 44	2	62	41	181	20	280 + 43
1905 Jan. 29	6 20	4 < 2	57	25	67	23	130 + 33
1905 Feb. 11	10 22	2	51	32	46	13	150 - 7
1905 Feb. 11	10 31¾	2	62	20	54	20	166 + 33
1905 Feb. 11	11 45	3 < 2	63	46	40	12	145 - 13
1901 Feb. 13	10 41	2	56	26	36	...	72 + 41
1906 Feb. 13	10 42	2	61	28	36	...	105 + 51
1910 Feb. 17	6 8½	3 < 2	88	46	45	15	72 + 43
1911 Feb. 19	9 22	1 - 2	70	49	590	14	46 - 15
1898 Feb. 20	8 54½	F	61	27	95	24	176 + 12

Observers.	Notes.
J. H. B., T. H. A.	Boötid.
A. S. H., W. E. B.	Boötid.
T. H. A., F. G. Whitley	Boötid.
C. Grover, H. Kirkbridge	Boötid.
W. E. B., W. F. D.	Boötid.
A. S. H., W. E. B. 10 observers	Boötid. A. S. H. found radiant $228^{\circ} + 50^{\circ}$. In full sunshine. Aquilid fireball.
G. F. Oldham, W. E. Rolston	Motion very slow. α Cetid.
J. G. Widdin, T. Harries	ϵ Arietid fireball.
C. Grover and 10 others	A bright Monocerotid.
17 observers	Left luminous trail of pale orange colour.
T. F. Bissel, H. E. Banting	Possibly α Boötid with Rad. $229 + 52$.
S. J. J. and 2 others	Motion very slow. λ Hydrid.
8 observers	Monocerotid. See Jan. 10, 14, and 27 for others.
T. Callard and 2 others	Same radiant as last and Jan. 10 and 27.
H. Corder, C. T. Marshall	Radiant a little uncertain.
33 observers	Long flight observed by many persons.
C. L. B., G. M. Knight, C. J. Lacey	Bright ζ Geminid.
A. S. H., J. H. B.	
4 observers	Bright ϵ Arietid.
J. H. Yard, J. L. Haughton	Bright Cepheid.
A. S. H., A. K.	α Canis Majorid.
5 observers	Very slow motion, compare Jan. 10 and 14.
14 observers	A very brilliant θ Boötid.
T. W. B. and 2 others	α Lynceid fireball. Compare Feb. 13.
A. S. H., J. H. B., T. W. B., and others	Prof. Herschel placed the radiant at
J. H. B., A. Parbury, and others	Cancerid fireball. [274 + 48.
H. Thomson and 3 others	λ Hydrid.
A. S. H. and 5 others	ξ Ursid.
16 observers	λ Hydrid fireball.
A. S. H. and another	A brilliant η Aurigid.
Miss M. A. Blagg, A. W. Harding, A. T. Poucher	A brilliant Lynceid.
12 observers	A fine Aurigid fireball.
10 observers	Very long flight.
18 observers	A β Leonid fireball.

Date.	G.M.T.	Mag.	App.	Dis.	Path.	Vel.	Radiant.
	h m		miles.	miles.	miles.	m.p.s.	° °
1909 Feb. 22	7 33	$\frac{1}{2} = \mathbb{D}$	56	50	155	25	196 + 20
1901 Feb. 27	8 52	F	103	57	120	...	165 + 6
1905 Feb. 28	10 15	$> \varphi$	73	43	61	slow	220 + 40
1910 Feb. 28	6 55	$6 \times \varphi$	82	16	145	25	155 + 16
1905 Feb. 28	12 10	$> \mathbb{D}$	69	28	94	slow	150 - 11
1899 Mar. 1	7 0	$4 \times \varphi$	73	28	53	21	119 + 31
1899 Mar. 1	8 49	$> \varphi$	58	22	39	10	119 + 33
1901 Mar. 6	7 25	F	84	27	66	...	51 + 43
1911 Mar. 9	7 57	Sirius	75	44	51	16	100 0
1909 Mar. 17	9 4	I	69	44	49	16	264 + 63
1905 Mar. 18	8 47	$\frac{1}{2} \mathbb{D}$	47	43	60	15	168 - 23
1906 Mar. 23	10 52	$\frac{1}{2} \mathbb{D}$	68	45	47	...	218 + 10
1910 Mar. 26	7 50	$\frac{1}{2} = \mathbb{D}$	63	48	126	24	262 + 39
1900 Mar. 28	10 33 $\frac{1}{2}$	2	54	50	13	29	255 + 21
1900 Mar. 28	10 45	2 - I	56	40	16	16	179 + 49
1900 Mar. 28	11 29	3 - 2	73	65	18	36	252 + 22
1902 Apr. 1	8 16	2 - I	70	41	29	19 $\frac{1}{2}$	140 + 50
1901 Apr. 1	9 44 $\frac{1}{2}$	F	69	12	67	...	129 + 25
1899 Apr. 4	7 59	$2 \times \varphi$	60	52	160	...	202 - 10
1898 Apr. 5	10 15	F	89	25	162	11	121 - 1
1903 Apr. 11	12 38	4 - 3	63	45	28	35	237 + 8
1906 Apr. 15	8 6	$3 \times \varphi$	65	28	39	18	153 + 33
1906 Apr. 16	8 53	$3 \times \varphi$	69	22	53	15	187 + 33
1898 Apr. 16	9 46	$3 - I\frac{1}{2}$	72	55	52	...	212 - 11
1911 Apr. 16	9 50	2	66	52	67	15	101 + 4
1898 Apr. 16	10 48	$I\frac{1}{4} - I$	60	48	91	24	189 - 31
1898 Apr. 17	9 40	$2\frac{1}{4} - I$	85	71	81	41	261 + 14
1898 Apr. 17	9 55	$4\frac{1}{2} - 3$	57	55	24	slow	236 - 13
1898 Apr. 17	10 28	5 - 3	72	70	34	23	176 - 35
1898 Apr. 17	11 28	2	65	61	25	36	291 + 13
1898 Apr. 17	11 12	$3 - 2\frac{1}{4}$	67	44	24	slow	195 + 40
1898 Apr. 18	11 55	$2 - I\frac{1}{2}$	65	58	19	32	274 + 11
1901 Apr. 18	13 19	3 - 1	83	55	35	40	266 + 33
1901 Apr. 18	14 47	2 - I	58	44	23	15	247 0
1903 Apr. 19	10 6 $\frac{1}{2}$	$I - \mathcal{U}$	59	26	65	18 $\frac{1}{2}$	192 - 6

Observers.

Notes.

200 observers	Coma Berenicid. Fireball ; streak visible 3 hours.
Alex. Spark, A. Cadenhead, and others	τ Leonid fireball.
Rev. J. Brown and others	β Boötid.
16 observers	A brilliant α Leonid fireball.
10 observers	Compare Jan. 13 and Feb. 11.
Sir W. J. Herschel and T. H. A.	} A pair of brilliant β Geminid fireballs.
T. H. A. and 3 others	
8 observers	$\alpha - \beta$ Perseid fireball.
H. Corder, H. Denning	ξ Draconid.
W. F. D. and others	
Rev. W. F. A. Ellison, W. Hilliard	Craterid.
W. Warner and 2 others	The radiant point and height are doubtful.
Mrs. Annie Warner and 6 others	A brilliant Herculid.
A. S. H., A. K., J. H. B.	Compare meteor 11 ^h 29 ^m same night.
A. S. H., A. K., J. H. B.	χ Ursid.
A. S. H., J. H. B.	δ Herculid.
A. K., T. H. A.	θ Ursid.
7 observers	Cancerid.
Rev. R. Killip, Parker, and Parry	A brilliant α Virginid.
T. E. R. P. and 4 others	Monocerotid.
A. S. H., W. F. D.	α Serpenticid.
Miss M. A. Blagg, J. S. Palmer, Joseph Hough	Leonid Minorid.
W. F. D., F. P. P., T. H. A., Miss Irene Warner	Canis Venaticid.
A. S. H., A. K.	α Virginid.
W. F. D., etc.	Monocerotid.
A. S. H., A. K.	Hydrid.
A. S. H., A. K.	η Librid.
A. S. H., A. K.	
A. S. H., W. F. D.	ϵ Sagittid.
A. S. H., A. K., W. F. D.	
A. K., W. F. D.	α Canis Venaticid.
A. S. H., W. F. D.	Ophiuchid.
A. S. H., W. F. D.	Very accurate path of an early Lyrid.
A. S. H., W. F. D.	λ Ophiuchid.
A. S. H., A. K.	Virginid.

Date.	G.M.T.	Mag.	App.	Dis.	Path.	Vel.	Radiant.
	h m		miles.	miles.	miles.	m.p.s.	° °
1907 Apr. 19	11 21	2 - 1	65	47	23	29	240 + 45
1909 Apr. 20	9 59	> ♀	86	56	90	R	270 + 30
1900 Apr. 20	11 44	1	70	63	53	35	210 - 31
1906 Apr. 21	13 4	Sirius	87	79	37	29	294 0
			89	56	32	25	275 + 35
1901 Apr. 21	9 29	3 × ♂	62	58	53	30	311 + 36
1900 Apr. 21	9 54	2	53	52	15	30	218 - 27
1900 Apr. 21	10 4	3 - 2	113	71	73	94	267 + 46
1901 Apr. 21	10 8	> 1 - ♀	79	51	68	40	268 + 30
1900 Apr. 21	10 32	3½ - 2	60	54	50	17	188 - 32
1900 Apr. 21	10 37	3	54	52	25	28	218 - 31
1901 Apr. 21	11 22	1	66	44	55	14	21 + 59
1903 Apr. 22	10 23	1 > 1	45	26	20	6½	150 + 49
1903 Apr. 22	10 36	1 - ♀	70	56	28	19	271 + 34
1908 Apr. 22	10 0	4	62	44	38	8	209 - 11
1903 Apr. 22	12 32½	1 - > ♀	78	43	49	39	269 + 32
1898 Apr. 22	10 30	5 - 4	72	52	26	slow	252 + 49
1903 Apr. 22	12 35½	4 - 2½	49	44	20	20	216 - 25
1897 Apr. 23	11 21	3 - 1	61	48	22	22	213 - 3
1897 Apr. 23	11 37	3 - 1	62	53	15	21	196 0
1909 Apr. 24	11 28	1	88	48	130	32	278 + 10
1897 Apr. 30	9 38	5 × ♀	59	18	70	23	291 + 59
1911 Apr. 30	11 58	3 × ♀	73	28	62	15	200 + 7
1902 May 2	14 27	1	48	44	17½	12	203 - 10
1900 May 3	13 58	5 - 2	54	49	155	28	337 0
1911 May 4	8 52	3 × ♀	76	41	87	28	254 + 25
1900 May 5	8 20	F	64	43	112	20	245 + 5
1897 May 5	12 31½	4 - 1	68	48	35	20	221 - 3
1897 May 5	12 49	4 - 2½	75	65	33	35	296 - 1
1910 May 10	7 52	D	83	32	100	17	73 + 42
1905 May 10	12 45	> 1	69	39	43	32	137 + 66
1911 May 16	10 0	1	55	55	220	18	30 + 38
1908 May 17	9 43	> ♀	76	28	57	15	194 + 20
1910 May 18	8 25	2 × ♀	76	55	34	...	233 + 8
1905 May 19	9 50	Sirius	56	49	80	slow	254 - 20

Observers.	Notes.
C.L.B., J.P. Kenyon	Herculid.
6 observers in France	Lyrid fireball.
A.S.H., A.K., J.H.B.	Hybrid.
C.L.B., J.P. Kenyon	{ Probably a bright Lyrid, but the recorded paths indicate a radiant near η Aquilæ.
W.F.D. and another	Bright ϵ Cygnid.
A.S.H., T.H.A.	Hybrid.
A.S.H., T.H.A., J.H.B.	Heights and velocity much above the average.
W.F.D., C.L.B., T.H.A.	A fine Lyrid.
A.S.H., A.K., J.H.B.	Hybrid.
A.S.H., J.H.B.	Hybrid.
C.L.B., W.F.D.	A well observed δ Cassiopeid.
C.L.B., A.K.	A bright very slow-moving meteor.
A.S.H., T.H.A.	Bright Lyrid.
W.F.D., etc.	A Virginid with very slow motion.
A.S.H., A.K.	Bright Lyrid.
A.S.H., W.F.D.	Herculid.
A.S.H., A.K.	Hybrid.
A.S.H., W.F.D.	
A.S.H., W.F.D.	Virginid.
W.F.D., etc.	Long flight.
W.F.D., C. Grover, and 5 others	A very fine α Draconid.
T.E.R.P. and 2 others	Observed nearly stationary.
A.S.H., J.H.B.	α Virginid.
A.S.H., J.H.B.	See <i>Note</i> at end of list.
S.A. Wilson and 7 others	A brilliant δ Herculid.
15 observers	Fireball from Ophiuchus.
W.F.D., H.R. Maynard	
A.S.H., W.F.D.	η Aquilid.
22 observers	In bright sunshine. η Aurigid.
A.S.H., G. Smith	Compare 1899 Oct. 5.
W.F.D. and others	Long flight. Observed path 118° .
H. Thomson, T.F. Donnelly, and others	Coma Berenicid.
F.W. Longbottom, etc.	
G. Smith, J. Williams	Scorpiid.

Date.	G.M.T.		Mag.	App. miles.	Dis. miles.	Path. miles.	Vel. m.p.s.	Radiant.	
	h	m						°	'
1908 May 19	10	20	> ♀	69	45	142	22	252	- 20
1903 May 23	11	58½	3 - 2½	59	47	14	23	290	+ 71
1910 May 24	11	0	> ♀	70	44	145	...	248	- 20
1911 May 24	10	38	3 - 2	72	53	20	35	243	+ 34
1903 May 26	11	46	1½ - 1	64	53	51	31	321	+ 11
1902 May 27	12	22½	1½ - ♀	63	39	120	24	282	- 24
1908 May 27	12	28	> ♀	58	55	120	15	76	+ 43
1911 May 29	10	42	3 - 1	74	57	91	30	350	+ 37
1911 May 31	10	50	> ♀	56	31	143	26	147	+ 4
1899 June 1	9	20	4 × ♀	...	30	...	...	209	+ 51
1911 June 1	10	22	4 - 1	65	56	65	44	355	+ 38
1910 June 1	9	37	♀	62	48	102	25	250	- 22
1899 June 2	8	47	♀	61	51	170	slow	250	- 23
1900 June 10	9	10	F	65	28	55	32	336	+ 73
1904 June 20	9	56	♀	66	42	42	...	302	+ 23
1908 June 25	10	43	2 - 1	61	59	51	30	43	+ 37
1908 June 28	11	12	> ♀	67	45	84	12	237	- 18
1899 June 29	10	33	3 - 1	66	42	49	...	330	+ 26
1908 July 1	9	15	½ = ♀	67	43	75	19	240	- 18
1899 July 4	10	50	2	54	46	27	13	281	- 16
1911 July 4	11	6	> ♀	79	56	105	26	48	+ 44
1899 July 8	10	42½	4	53	41	14	18	316	+ 48
1899 July 8	10	55	3 - 1	46	42	14	10	253	- 20
1899 July 8	10	57	4 - 2	36	33	3	...	294	+ 61
1901 July 8	11	37	3 - ♀	53	26	151	12	253	- 24
1904 July 12	11	57	3 - 2¼	69	51	24	27	304	+ 5
1904 July 13	10	55	3	75	69	15	21	283	- 13
1902 July 13	10	30	1	89	51	51	26	315	+ 31
1902 July 13	12	1	2 - 1	63	47	21	21	323	+ 22
1902 July 15	9	32	♀ - ♀	71	55	53	18	227	- 17
1900 July 15	10	13	2 × ♀	51	21	78	16	297	- 11
1899 July 16	11	10	4	78	65	14	18	263	+ 36
1898 July 16	11	42	2 > 1	79	37	46	21	273	+ 76
1900 July 17	8	47	F	58	15	175	...	249	- 20
1904 July 17	11	22½	3	84	52	43	48	339	+ 36

Observers.	Notes.
Rev. W.F.A. Ellison and J.M'Harg	Brilliant Scorpiid.
A.S.H., W.F.D.	δ Draconid.
E.M. Anderson and another	Scorpiid.
Mr. and Mrs. W., W.F.D.	Coronid.
A.S.H., W.F.D.	ϵ Pegasid.
A.S.H., E.H. Cullum	α Sagittarid.
A.A. Nijland, C. Birkenstock	Brilliant α Aurigid.
Mr. and Mrs. W., W.F.D.	See June 1, 10 ^h 22 ^m .
J.P. Davies and 3 others	From Sextans.
3 observers	Imperfectly recorded.
Mr. and Mrs. W., W.F.D.	Compare May 29.
10 observers	Scorpiid.
H.C., T. Harries and others.	Scorpiid.
9 observers	β Cepheid fireball.
C.S. Russell, J. Webb	The radiant is doubtful.
W.F.D., T.H.A.	Early β Perseid.
W.F.D., Rev. J. Brown, A.H.S. Palmer, and others	Librid fireball.
T.H.A., J.H.B.	Pegasid.
6 observers	Librid fireball.
T.H.A., J.H.B., W.F.D.	λ Aquilid.
Mrs F. Edwards, W.G. Royal-Dawson	
A.S.H., J.H.B.	α Cygnid.
A.S.H., J.H.B., W.F.D.	Scorpiid. Motion very slow.
A.S.H., T.H.A.	σ Draconid. Path only 3 miles!
Miss L.M. Milner, H.M. Whitley	Scorpiid.
A.S.H., W.F.D.	Aquilid.
T.H.A., J.H.B.	λ Aquilid.
W.F.D. and 60 others	Brilliant Cygnid, flashed like lightning.
A.S.H., W.F.D.	A well-observed Vulpeculid.
A.S.H. and 5 others	A.S.H. places radiant at $236^{\circ} - 12^{\circ}$.
W.F.D. and 3 others	A brilliant Capricornid.
A.S.H., W.F.D.	ρ Herculid.
A.K., W.F.D.	ψ Draconid.
17 observers	A brilliant Scorpiid. In evening twilight.
C.L.B., A.K.	η Pegasid.

Date.	G.M.T. h m	Mag.	App. miles.	Dis. miles.	Path. miles.	Vel. m.p.s.	Radiant. ° °
1900 July 18	11 33½	4	52	50	45	11	214-10
1908 July 19	10 43	2	64	29	35	24	278+57
1900 July 19	11 49	1-4	81	54	43	35	17+50
1908 July 22	10 57½	2½-2	74	53	47	19	300-11
1911 July 22	10 40	4-2	48	26	25	12	335+50
1900 July 23	11 13	2-1	83	59	49	30	30+47
1900 July 23	12 12	2-1	84	55	44	59	24+52
1900 July 23	13 1	2-1½	67	66	51	...	54+9
1900 July 24	10 42	3-2	72	63	28	12	305-17
1900 July 24	10 49	3×♀	68	27	103	19	280-15
1903 July 24	11 28	2>1	68	43	44	73	51+65
1900 July 24	12 17½	4-1	78	71	39	49	14-6
1900 July 25	11 10	3-1	75	48	46	swift	8+39
1908 July 25	12 30	2	69	41	30	18	280+59
1898 July 26	9 10	½=D	73	27	191	slow	269-23
1903 July 26	12 35	2-1	66	46	29	53	358+27
1903 July 26	12 44¾	3	51	39	21	6½	233+38
1911 July 26	10 55	>1	91	63	35	44	331+62
1902 July 27	10 28	4-1½	70	54	32	swift	30+54
1902 July 27	11 28	4-3	67	50	20	24	283+24
1902 July 27	11 36½	4	71	52	45	33	351+3
1902 July 28	11 7	2	74	71	43	30	44+21
1900 July 28	11 46	3-1	65	44	52	39	337-10
1908 July 28	11 6	=D	82	40	50	34	302+23
1911 July 28	11 4	>1	62	49	64	8½	254-22
1900 July 28	12 35	4-3	84	76	32	40	16-5
1899 July 29	11 9	4	55	54	12	13	317-31
1899 July 29	11 13	2>1	62	47	17	27	323+34
1902 July 29	11 27	2-4	57	47	18	18	305-5
1899 July 29	11 51	4	68	57	25	33	3+10
1898 July 30	10 43	1-4	81	47	57	36	23+53
1900 July 30	10 45	F	95	50	92	swift	30+52
1898 July 30	10 53	4-3½	61	52	20	14	311-10
1898 July 30	11 10	3½-3	56	50	28	20	340-18
1897 July 30	8 47	>♀	54	23	50	13	155+59

Observers.	Notes.
3 observers	A.S.H. gives radiant at $209^{\circ} - 8^{\circ}$.
W.F.D., T.K.Jenkins	Draconid.
A.S.H., W.F.D.	Early Perseid.
W.F.D., J.P.Kenyon	α Capricornid.
Mr. and Mrs. W., W.F.D.	Lacertid.
A.K., W.F.D.	? Perseid.
A.S.H., W.E.B., W.F.D.	Perseid. A.S.H. found the radiant at $20^{\circ} + 51$.
W.E.B., W.F.D.	A very early ϵ Taurid.
A.K., W.F.D.	α Capricornid.
W.F.D. and 4 others	A brilliant λ Aquilid. Compare July 13.
A.K., W.F.D.	Apparently very rapid.
W.E.B., W.F.D.	Compare July 28, 12 ^h 35 ^m .
A.S.H., W.F.D.	μ Andromedid.
W.F.D., T.K.Jenkins	Draconid.
F.C.Dennett, C.Grover, W.F.Kelsey	δ Sagittarid fireball.
A.S.H., W.F.D.	α Andromedid.
A.K., W.F.D., A.S.H.	Velocity exceedingly slow.
Mr. and Mrs. W., W.F.D.	Bright α Cepheid.
A.K., T.H.A.	Perseid. A.S.H. gives radiant at $27^{\circ} + 53$.
A.S.H., C.L.B.	
A.K., P.M.Ryves	
A.K., W.F.D.	An early ϵ Arietid.
A.S.H., W.F.D.	Aquarid.
Rev. W.F.A.Ellison and 10 others	Sagittid. Fine flashing fireball.
W.F.D. and others	Scorpiid with very slow motion.
A.S.H., W.F.D.	Compare July 24, 1904.
A.S.H., W.F.D.	Radiant on W. border Piscis Australis.
A.S.H., W.F.D.	ζ Cygnid.
A.K., C.L.B.	α Capricornid.
A.S.H., W.F.D.	γ Pegasid.
A.S.H., W.F.D.	Perseid.
5 observers	Perseid fireball.
A.S.H., W.F.D.	μ Aquarid.
A.S.H., W.F.D.	Erratic Aquarid.
7 observers	Precise radiant uncertain.

Date.	G.M.T.	Mag.	App.	Dis.	Path.	Vel.	Radiant.
	h m		miles.	miles.	miles.	m.p.s	° °
1898 July 30	11 35½	3 - 1	66	52	63	45	42 + 23
1900 July 30	11 45	3 - 2	56	40	52	35	338 - 10
1911 July 30	10 24	2	55	36	49	slow	308 - 12
1911 July 30	10 27	¼ - ½	85	51	66	swift	30 + 54
1911 July 31	8 17½	½	61	40	92	slow	309 - 12
1911 July 31	10 34	2	72	49	66	18	306 - 10
1897 Aug. 1	10 39	4	31	16	21	24	357 + 42
1899 Aug. 1	11 10½	3 - 2	68	50	59	...	335 - 11
1897 Aug. 1	12 14	2 - 1	55	42	25	...	296 - 6
1911 Aug. 1	11 49	3 × ♀	103	53	65	43	34 + 54
1911 Aug. 1	11 1	3 - 2	51	44	19	8	309 - 16
1897 Aug. 2	10 46	3 - 2	44	40	7	swift	110 + 70
1897 Aug. 2	10 52	3½	59	55	44	15	339 - 24
1897 Aug. 2	11 5½	2	112	90	40	swift	40 + 55
1897 Aug. 2	11 24	5 - 4	139	124	28	swift	73 + 66
1897 Aug. 2	13 57½	3 × ♀	81	47	59	36	312 0
1908 Aug. 2	10 26	> 1	82	54	50	25	39 + 56
1908 Aug. 2	10 30	> 1 - ♀	77	47	57	38	38 + 54
1908 Aug. 2	12 7	1 - 2	61	49	32	21	337 - 11
1908 Aug. 2	11 2	1	79	55	40	40	34 + 54
1897 Aug. 3	13 40	2 - 1½	60	54	61	37	72 + 11
1908 Aug. 3	10 32	3	70	44	26	18	270 + 49
1906 Aug. 5	10 33	> ♀	65	52	24	30	38 + 55
1905 Aug. 5	10 56	3 - 1	56	47	43	9½	304 - 25
1899 Aug. 6	10 36	2½ - 2	78	60	33	33	39 + 57
1908 Aug. 6	10 24½	2	66	50	30	30	40 + 57
1899 Aug. 8	9 15	> ♀	133	115	63	42	52 + 47
1909 Aug. 8	10 8	2	94	65	58	48	41 + 57
1904 Aug. 9	10 17½	1½ - ♀	69	52	32	swift	44 + 59
1899 Aug. 9	10 19	3 - 2	88	61	51	swift	40 + 57
1897 Aug. 9	13 27	3 - 1	140	77	81	100	46 + 56
1897 Aug. 9	13 52	3 - 3	131	89	56	swift	58 + 60
1897 Aug. 9	14 18	3 × ♀	137	75	75	swift	44 + 45
1899 Aug. 10	10 14	♀	76	40	54	swift	41 + 57
1899 Aug. 10	10 54	2	97	51	79	swift	45 + 58

Observers.	Notes.
A.S.H., W.F.D.	An early ϵ Arietid.
A.S.H., W.F.D.	Aquarid.
Mrs. Freeman, W.A. Robson	α Capricornid.
D. Booth, S.D. Wall, and others	A brilliant Perseid.
12 observers	A brilliant α Capricornid.
W.F.D., etc.	A brilliant α Capricornid.
A.S.H., W.F.D.	Unusually low.
J.H.B., W.F.D.	Aquarid.
A.S.H., W.F.D.	Aquilid.
Mr. and Mrs. W., W.F.D.	Perseid fireball.
Mr. and Mrs. W., W.F.D.	α Capricornid.
H.C., W.F.D.	Extremely short path.
A.S.H., H.C., W.F.D.	Piscid Australid.
H.C., W.F.D.	Perseid. Great elevation.
H.C., W.F.D.	Camelopardid. Great elevation.
A.S.H., H.C.	
W.F.D., T.K. Jenkins	Perseid. Slow velocity.
Miss Warner, G. Powell	Brilliant Perseid.
C.L.B., W.F.D.	Aquarid.
C.L.B., W.F.D.	Perseid.
A.S.H., W.E.B.	$\circ$ Orionid.
W.F.D., T.K. Jenkins	γ Draconid.
W.F.D., Miss Warner, and others	Brilliant Perseid.
A.S.H., G. Smith, W.F.D.	Very slow motion.
T.H.A., W.F.D.	Perseid.
W.F.D., T.K. Jenkins	Perseid.
A.S.H., N. Lattey	Perseid. Height abnormal.
W.F.D., T.K. Jenkins	Bright Perseid.
T.H.A., J.P. Kenyon	Bright Perseid.
T.H.A., T.E.R.P., W.F.D.	Perseid.
A.S.H., W.E.B.	Perseid. Height abnormal.
A.S.H., W.E.B.	Perseid. Height abnormal.
A.S.H., W.E.B.	Perseid. Height abnormal.
T.H.A., T.E.R.P., W.F.D., S.J.J.	Brilliant Perseid.
T.H.A., T.E.R.P., W.F.D., S.J.J.	Bright Perseid.

Date.	G.M.T.	Mag.	App.	Dis.	Path.	Vel.	Radiant.
	h m		miles.	miles.	miles.	m.p.s.	° °
1902 Aug. 10	10 8 $\frac{1}{4}$	3 - 1 $\frac{1}{2}$	62	53	25	9	197 + 31
1902 Aug. 10	10 15 $\frac{1}{4}$	2	73	66	26	47	80 + 47
1902 Aug. 10	10 26 $\frac{1}{4}$	3 - 1	65	46	20	100	305 + 54
1909 Aug. 10	11 15	4	87	52	62	40	43 + 56
1901 Aug. 10	10 41	3 - 2	88	68	21	25	278 + 67
1901 Aug. 10	10 58	3 - 2	57	52	25	15	352 - 14
1901 Aug. 10	12 0 $\frac{1}{2}$	1	76	51	36	50	44 + 58 $\frac{1}{2}$
1901 Aug. 10	12 16	2 - 1	69	50	37	39	42 + 59
1901 Aug. 10	12 19	3 - 2	72	60	33	25	149 + 60
1901 Aug. 10	13 18	1 - 4	70	47	31	31	45 + 56
1901 Aug. 10	13 34	> 1	63	29	48	16	269 + 47
1908 Aug. 10	11 38	1	90	51	61	very swift	44 + 57
1908 Aug. 10	10 17	4	72	45	52	29	43 + 57
1908 Aug. 10	10 48	2	88	56	54	40	45 + 56
1910 Aug. 10	10 6	> 9	75	48	54	27	45 + 56
1910 Aug. 10	12 6	4	81	51	42	30	43 + 56
1898 Aug. 11	8 58	4 - 9	66	41	196	slow	339 - 10
1898 Aug. 11	10 15 $\frac{1}{2}$	4 - 9	92	68	45	swift	44 + 56
1898 Aug. 11	10 26 $\frac{1}{2}$	2 $\frac{1}{2}$ - 2	86	61	45	58	46 + 58
1904 Aug. 11	10 39	1 - 9	75	56	33	35	46 + 58
1898 Aug. 11	10 46	2 $\frac{1}{2}$ - 4	79	65	23	46	46 + 58
1901 Aug. 11	11 2	4 - 9	95	56	64	swift	45 + 58
1899 Aug. 11	11 3 $\frac{1}{2}$	1	75	53	38	38	46 + 57
1898 Aug. 11	11 11 $\frac{1}{2}$	1 > 1	85	49	57	swift	44 + 56
1897 Aug. 11	11 15	1 > 1	79	57	37	swift	47 + 55
1901 Aug. 11	11 33	4	94	59	55	42	45 + 57
1898 Aug. 11	11 52	2 - 1 $\frac{1}{2}$	55	49	53	27	4 - 20
1898 Aug. 11	11 57	3 2	56	39	25	36	57 + 73
1898 Aug. 11	12 6 $\frac{1}{2}$	1	86	46	59	swift	46 + 58
1898 Aug. 11	12 16 $\frac{1}{2}$	1 $\frac{1}{2}$ - 1	74	50	33	swift	50 + 63
1898 Aug. 11	12 20	1 $\frac{1}{2}$ - 1	64	48	24	37	54 + 56
1898 Aug. 11	12 24 $\frac{1}{2}$	> 1 - 4	78	52	36	38	49 + 59
1898 Aug. 11	12 40	2 $\frac{1}{2}$ - 1	85	60	32	31	43 + 58
1898 Aug. 11	12 46 $\frac{1}{2}$	3 - 2 $\frac{1}{2}$	75	59	21	swift	59 + 63
1904 Aug. 11	13 5 $\frac{1}{2}$	9	74	50	30	swift	45 + 58

Observers.	Notes.
A.K., C.L.B.	Slow motion.
A.K., C.L.B.	α Aurigid.
A.K., T.H.A.	Cygnid. Velocity abnormal.
W.F.D. and others	Bright Perseid.
A.S.H., W.F.D.	
A.S.H., J.H.B., W.F.D.	ω Aquarid (see Aug. 20, 12 ^h).
A.S.H., W.F.D.	Perseid.
A.S.H., W.F.D.	Perseid.
A.S.H., W.F.D.	Ursid.
O.A. LeBeau, W.F.D.	Bright Perseid.
O.A. LeBeau, W.F.D.	γ Draconid.
Greenwich, W.F.D.	Perseid.
J.P. Kenyon, W.F.D.	Bright Perseid.
W.F.D., Miss Warner, G. Powell	Perseid.
T.K. Jenkins and W.F.D.	Bright Perseid.
C.L.B. and another	Bright Perseid.
Col. J. Herschel, W.F.D., and others	Aquarid fireball. Long flight.
W.E.B., W.F.D.	Perseid fireball.
A.S.H., W.F.D.	Perseid.
S.J.J., W.F.D.	Bright Perseid.
A.S.H., W.F.D.	Perseid.
W.F.D., D.E. Packer, F.L. Raymond	Bright Perseid.
A.S.H., W.F.D.	Bright Perseid.
W.E.B., W.F.D.	Bright Perseid.
J.A. Hardcastle, W.F.D.	Bright Perseid.
D.E. Packer, W.F.D.	Bright Perseid.
A.K., W.F.D.	Perseid.
A.S.H., A.K.	γ Camelopardid.
T.E.R.P., S.J.J.	Perseid.
A.S.H., W.F.D.	Erratic Perseid.
A.S.H., W.F.D.	Perseid.
A.S.H., W.F.D.	Bright Perseid.
A.S.H., W.F.D.	Perseid.
A.S.H., W.F.D.	Erratic Perseid.
W.E.B., P.M. Ryves	Bright Perseid.

Date.	G.M.T. h m	Mag.	App.	Dis.	Plate.	Vel.	Radiant.	
			miles.	miles.	miles.	m.p.s.	°	°
1907 Aug. 11	13 16	3 × ♀	86	50	42	swift	45	58
1908 Aug. 11	9 40	> 4	99	52	93	46	46	59
1909 Aug. 11	9 57	4	103	48	110	swift	43	58
1909 Aug. 11	10 5	1 > 1	78	56	41	R	56	65
1909 Aug. 11	10 15	1 - 4	78	51	58	46	45	56
1909 Aug. 11	10 40	Sirius	86	55	63	37	47	59
1909 Aug. 11	11 19	1 - ♀	98	50	65	43	50	58
1903 Aug. 12	10 55	4	67	65	50	32	32	4
1907 Aug. 12	11 10½	♀ - D	79	52	34	swift	41	57
1909 Aug. 12	9 42	3 × ♀	87	53	68	35	47	58
1909 Aug. 12	9 51	1 - 4	90	62	58	46	47	58
1909 Aug. 12	10 19	4	78	52	52	30	47	58
1909 Aug. 12	10 42	1	83	51	51	40	38	60
1910 Aug. 12	11 49	4	89	53	51	40	9	27
1899 Aug. 14	9 31	3 - 1	67	61	19	swift	52	46
1911 Aug. 14	9 18	4	75	46	56	38	50	59
1899 Aug. 15	8 37½	2 - 1	73	48	27	...	334	74
1901 Aug. 15	10 6½	4 - 3	77	58	38	60	56	58
1901 Aug. 15	10 48	4 - ♀	81	61	37	41	52	53
1901 Aug. 17	10 12	3 - 2	72	66	27	45	58	34
1901 Aug. 17	11 5	2½ - 2	58	53	22½	32	95	46
1898 Aug. 17	11 43	2 - 1	71	55	32	43	72	52½
1901 Aug. 18	13 24½	4 - 3	62	50	28	15	106	52
1900 Aug. 19	10 36	3 × ♀	56	29	62	...	346	0
1901 Aug. 19	13 41½	2	64	50	18	28	337	3
1901 Aug. 20	11 8	1 > 1	110	71	40	...	329	61
1901 Aug. 20	11 14	> 4	56	33	44	13	213	49
1901 Aug. 20	12 0	2	64	58	17	23	347	15
1898 Aug. 21	9 16	1½ × ♀	60	29	95	19	5	13
1901 Aug. 21	10 19	2 > 1	63	52	20	25	341	5
1901 Aug. 21	11 7½	2 - 1	71	60	34	40	51	27
1903 Aug. 21	12 43	4 - 1	64	46	21	40	1	25
1902 Aug. 21	14 2	= D	65	33	611	15½	283	10½
1900 Aug. 22	10 8	2 - 4	75	50	25	25	310	53
1899 Aug. 24	8 11	F	77	27	118	17	345	14

Observers.	Notes.
W.F.D., Greenwich, and others	Perseid fireball.
E. Hawks, H. Denning	Bright Perseid.
Miss Warner, T.K. Jenkins	Bright Perseid.
Mrs. R.M. Brook, J.P. Kenyon	Erratic Perseid.
W.F.D., J.P. Kenyon	Perseid.
W.F.D., J.P. Kenyon	Perseid.
Miss Warner, C.B. Pennington	Perseid.
W.F.D., S.J.J.	Bright Arietid.
Miss Warner, H.C., and 2 others	Perseid fireball.
W.F.D. and 35 others	Perseid fireball.
W.F.D., G. Powell	Bright Perseid.
W.F.D., G. Powell, J.P. Kenyon	Bright Perseid.
W.F.D. and others	Erratic Perseid.
W.F.D., G. Powell	Bright α Andromedid.
J.H.B., I.F.H. Carr-Gregg	$\alpha - \beta$ Perseid.
Mr. and Mrs. W., W.F.D.	Late Perseid.
T.H.A., S.J.J.	β Cepheid.
J.H.B., W.F.D.	Late Perseid.
J.H.B., A.K.	Perseid fireball.
A.S.H., W.F.D.	ϵ Perseid.
A.S.H., C.L.B.	Lyncid.
A.S.H., W.F.D.	Camelopardid.
A.S.H., W.F.D.	Motion abnormally slow.
T.E.R.P., W.F.D.	β Piscid.
A.S.H., W.F.D.	ζ Aquarid.
P.M. Ryves, W.F.D.	α Cepheid.
C.J. Spencer, W.F.D.	Spencer's observation by photography.
A.S.H., W.F.D.	ω Aquarid.
A.S.H., A.R. Schutz, and others	γ Pegasid.
W.F.D., A.K.	
C.L.B., W.F.D.	Radiant W. of Pleiades.
A.K., W.F.D.	α Andromedid.
25 observers	Exceedingly long flight.
A.K., W.F.D.	Cygnid.
40 observers	α Pegasid.

Date.	G.M.T. h m	Mag.	App. miles.	Dis. miles.	Path. miles.	Vel. m.p.s.	Radiant. ° °
1910 Aug. 24	8 55	2	61	26	35	24	288 + 54
1902 Aug. 25	10 50	2	65	57	59	11½	339 - 26
1903 Aug. 25	9 35	3	70	52	42	52	79 + 59
1899 Aug. 27	10 13	> ♀	68	47	143	25	191 + 32
1906 Sept. 1	10 0	> 1	79	40	52	...	{ 314 + 10 320 + 12
1900 Sept. 2	6 54	F	85	20	84	...	334 + 57
1899 Sept. 2	12 4½	3 × ♀	86	60	28	...	46 + 42
1910 Sept. 2	9 5	> ♀	98	44	352	40	87 + 41
1911 Sept. 2	3 45	> D	72	49	46	swift	Leo - Virgo
1911 Sept. 2	8 27½	D	88	26	196	19	348 + 3
1907 Sept. 3	10 10	½ D	61	31	30	slow	290 + 61
1911 Sept. 3	9 59½	1½	60	43	29	10	335 0
1899 Sept. 4	8 13½	2 - ♀	67	21	52	...	311 + 79
1902 Sept. 7	11 34	3 - 2	82	66	18	31	28 + 68
1899 Sept. 8	8 21	> ♀	71	26	114	...	347 + 3
1911 Sept. 10	8 13½	1	66	60	40	16	140 + 40
1901 Sept. 14	8 45	2 > ♂	66	26	83	20	345 + 1
1908 Sept. 14	8 48	3 × ♀	59	32	71	...	6 + 7
1898 Sept. 15	8 17	1 - 2	69	61	58	39	65 + 35
1906 Sept. 15	10 24	3 × 2	77	43	72	20½	245 + 30
1900 Sept. 16	8 44	2 × ♀	50	32	86	...	324 - 25
1898 Sept. 16	10 55½	> 2 > 1	82	55	39	...	351 + 7
1898 Sept. 17	11 40¼	1 - 2	95	64	93	...	57 + 7
1898 Sept. 17	11 52	4½ - 1	64	52	19	19	343 0
1898 Sept. 18	8 54	> 2	69	46	104	very swift	84 + 41
1904 Sept. 18	12 10	> 2	58	23	53	21	13 + 7
1911 Sept. 18	10 22	> 1	92	43	110	24	62 + 34
1911 Sept. 18	9 14	4 - 3	48	44	17	8½	298 - 22
1911 Sept. 18	9 40	2½	71	29	43	swift	323 + 39
1910 Sept. 19	10 56	2	102	56	174	...	66 + 20
1903 Sept. 20	11 4½	4	51	42	16	8	319 - 4
1901 Sept. 21	11 22	4 - 1	72	54	24	...	27 + 22
1898 Sept. 22	9 13	3 - 1	107	82	34	...	254 + 39
1910 Sept. 24	9 56	> 2	68	38	48	24	192 + 70
1911 Sept. 24	8 10	3	72	61	12	swift	192 + 67

Observers.	Notes.
Mr. and Mrs. W., W.F.D.	Bright χ Cygnid.
C.L.B., W.F.D.	A well-observed Piscid Australid.
C.L.B., W.F.D.	β Camelopardid.
6 observers	Coma Berenicid. See Aug. 10, 10 8 $\frac{1}{2}$.
} Col. E.E. Markwick, C.D.R. Andrews	Exact radiant uncertain.
37 observers	A.S.H., 85 to 18 miles.
W.F.D., I.F.H. Carr-Gregg	A brilliant $\alpha - \beta$ Perseid.
11 observers	A fine β Aurigid.
7 observers	In brilliant sunshine.
Over 300 observers	β Piscid fireball.
W.L. Fox, C. James	σ Draconid. Lit up sky.
Mr. and Mrs. W., W.F.D.	ζ Aquarid.
T.H.A., A.S. Williams	Brilliant κ Cepheid.
A.S.H., W.F.D.	
T.H.A., A.K.	Same radiant as next.
Mr. and Mrs. W., W.F.D.	Lyncid.
20 observers	β Piscid.
A.K., D.E. Packer	γ Pegasid.
Col. J. Herschel, W.F.D.	A bright ϵ Perseid.
C.L.B., W.F.D.	ξ Coronid.
4 observers	ζ Capricornid.
A.K., W.F.D.	θ Piscid.
A.S.H., A.K.	e Taurid.
A.S.H., A.K.	β Piscid.
T.E.R.P. and another	Brilliant β Aurigid.
C.L.B. and 2 others	ϵ Piscid.
Mr. and Mrs. W., Mrs. Freeman	ϵ Perseid.
Mr. and Mrs. W., W.F.D.	Sagittarid.
Mr. and Mrs. W., W.F.D.	
Mr. and Mrs. W., G.H. Hunt, and others	Taurid.
A.S.H., W.F.D.	Very slow motion.
A.K., C.L.B.	α Arietid.
A.K., W.F.D.	η Herculid.
W.E. Yerward-James, F.T. Naish	κ Draconid.
Mr. and Mrs. W., W.F.D.	κ Draconid.

Date.	G.M.T.	Mag.	App.	Dis.	Path.	Vel.	Radiant.
	h m		miles.	miles.	miles.	m.p.s.	° °
1902 Sept. 25	7 34	> ♀	88	57	34	100	254 + 46
1897 Sept. 26	11 50	4	41	35	6½	slow	36 + 43
1906 Sept. 27	6 59	2 - ♀	63	45	46	...	345 + 2
1910 Sept. 29	12 35	♀	90	53	193	...	111 + 22
1911 Sept. 30	8 20	4 - 2	66	59	45	30	106 + 47
1911 Sept. 30	8 40	2½	79	52	34	16	4 + 28
1911 Sept. 30	9 7	♀	81	48	41	...	6 + 27
1911 Sept. 30	9 20	1½ - 2	69	69	54	20	119 + 34
1899 Oct. 2	8 24	♀	67	45	88	13	284 - 17
1899 Oct. 5	10 55½	2	52	45	14	20	137 + 65
1909 Oct. 6	9 33	2	75	30	65	15	{ 167 + 3 150 + 5
1911 Oct. 8	6 53	2	68	45	65	11	
1904 Oct. 8	7 0	1 - 1½ 2	66	25	82	10	261 + 4
1899 Oct. 10	11 6	3 - 1	48	43	13	21	35 - 10
1910 Oct. 10	9 15	2	95	46	94	...	77 + 32
1911 Oct. 11	6 35	1	85	43	63	25	335 + 28
1908 Oct. 14	9 10	> ♀	70	18	96	...	42 + 20
1902 Oct. 15	7 47	♀	62	54	130	37	150 + 43
1901 Oct. 20	11 40	3 - 2½	78	46	41	26	38 + 12
1900 Oct. 21	8 35	F.	68	20	66	11	300 + 22
1900 Oct. 21	11 58	♀	63	48	37	25	350 - 4
1900 Oct. 22	6 23	> ♀	59	35	171	...	218 + 10
1906 Oct. 22	12 16¼	3 - 1½	75	56	44	36	98 + 14
1908 Oct. 23	12 19	> ♀	73	47	33	...	33 + 16
1903 Oct. 23	11 47	1 - 2	75	37	48	21	40 + 16
1900 Oct. 23	12 29	3	96	81	31	44	99 + 13
1900 Oct. 23	12 30	2	87	52	52	35	74 + 14
1910 Oct. 23	8 13	2 × ♀	84	40	75	18	33 + 20
1900 Oct. 27	9 51	4 - 2	73	60	58	40	169 + 49
1900 Oct. 27	11 42	3 × ♀	76	67	34	42	136 + 34
1907 Oct. 31	10 0½	= 2	78	48	115	20	47 - 11
1910 Nov. 1	7 40	2	64	47	62	20	57 + 15
1910 Nov. 2	7 46	> ♀	85	26	160	23	41 + 14
1904 Nov. 3	10 55	2½ ♀	70	23	73	14	344 + 24
1911 Nov. 5	9 39	3 - 2	83	64	38	26	166 + 71

Observers.	Notes.
Rev. W.F.A. Ellison, W.F.D.	A very rapid flight.
A.S.H., J.A. Harcastle	
Miss Warner, J.H.P. Case	Brilliant β Piscid.
Val. Jackson and 2 others	δ Geminid.
Mr. and Mrs. W., W.F.D.	Lyncid.
Mr. and Mrs. W., W.F.D.	α Andromedid.
H.C., J.M'Dermott, etc.	Brilliant α Andromedid.
Mr. and Mrs. W., W.F.D.	β Geminid.
Col. J.H. Boileau, M.J. Nyhan	Brilliant λ Aquilid.
A.S.H., T.H.A.	Ursid.
} 22 observers	In bright sunshine.
W.F.D. and 2 others	Brilliant α Capricornid.
C.L.B. and others	Late β Ophiuchid.
A.S.H., J.H.B.	ϵ Cetid.
At Norfolk and Croydon	ι Aurigid.
M. Quennevais and another	η Pegasid.
S.H.R. Salmon and 2 others	ϵ Arietid.
A.K. and 2 others	μ Ursæ Majorid.
A.S.H., W.F.D.	μ Cetid.
300 observers	Very brilliant Sagittid.
A.S.H., C.L.B.	
T.R. Clapham and 4 others	Radiant point doubtful.
C.L.B., J.P. Kenyon	ζ Geminid.
2 observers	Seen in Germany.
C.L.B., W.F.D.	ϵ Ariel.
A.S.H., W.F.D.	ζ Geminid.
A.S.H., W.F.D.	
J.E. Clark and 3 others	Arietid fireball.
A.S.H., J.H.B.	ψ Ursæ Majorid.
Mrs. W.H.S. Monck, W.F.D.	Left streak 13 mins.
17 observers	Brilliant Eridanid.
E.J. Webb, H. Denning, etc.	λ Taurid.
Rev. W.F.A. Ellison and others.	Arietid.
A.S.H., and others	β Pegassid fireball.
Mr. and Mrs. W., J.B. Snell	λ Draconid.

Date.	G.M.T. h m	Mag.	App. miles.	Dis. miles.	Path. miles.	Vel. m.p.s.	Radiant. ° ° °
1910 Nov. 6	6 52	♀	85	51	190	30	57+15
1910 Nov. 9	10 24	♂	62	32	84	12	312+11
1910 Nov. 9	6 46	I	60	42	112	27	57+15
1910 Nov. 9	11 23	> I	88	55	52	17	68+ 6
1909 Nov. 9	7 15	♀	59	45	56	35	58+ 9
1901 Nov. 10	6 45	$\frac{1}{2}=D$	64	55	73	24	55+ 7
1899 Nov. 10	13 16	4-2	56	48	18	18	55+ 5
1899 Nov. 10	13 37 $\frac{1}{2}$	4	90	48	46	32	72+32
1901 Nov. 13	6 53	D	90	52	165	33	87+34
1900 Nov. 13	10 17	3 $\frac{1}{2}$ -I	68	57	18	25	58+ 7
1907 Nov. 13	11 13	I	69	46	52	15	284+57
1900 Nov. 13	13 32	2	69	58	24	48	151+22
1900 Nov. 13	13 58	2 $\frac{1}{2}$ -I $\frac{1}{2}$	85	58	33	47	145+29
1897 Nov. 13	15 28	I	125	77	75	...	136+ 9
1897 Nov. 13	15 52	I	103	59	60	...	152+22
1897 Nov. 13	16 47 $\frac{1}{2}$	3	52	40	22	37	176+10
1897 Nov. 14	10 26	I	83	78	157	40	150+24
1897 Nov. 14	14 37	2	79	58	35	39	151+23
1897 Nov. 14	16 25	> I > ♂	88	44	59	46	151+22
1899 Nov. 14	17 40	♂	71	42	46	...	193+27
1903 Nov. 14	10 46	> I	71	64	103	51	149+24
1901 Nov. 14	13 37 $\frac{1}{2}$	2-I	77	57	38	very swift	152+30
1901 Nov. 14	13 32	I-I	85	52	66	very swift	156+32
1901 Nov. 14	13 42	♂-♀	81	67	67	55	174+20
1901 Nov. 14	14 24	4-2	82	57	44	very swift	152+25
1901 Nov. 14	16 7	I	86	60	35	...	151+23
1901 Nov. 14	14 38	2	67	54	22	37	152+23
1901 Nov. 14	15 7	3-I	95	55	60	very swift	152+23
1901 Nov. 14	15 23	> I-4 ♂	72	47	36	72	151+25
1901 Nov. 14	16 0	> I	91	61	41	54	149+20
1903 Nov. 15	10 18	♂-♀	79	42	48	36	56+22
1901 Nov. 15	13 8	I $\frac{1}{2}$ -I	43	37	28 $\frac{1}{2}$	7 $\frac{3}{4}$	38-21
1903 Nov. 15	13 18	2	73	58	36	48	151+24
1903 Nov. 15	13 42	> ♀	72	32	47	23	61+24
1901 Nov. 15	13 48	2-♂	76	59	37	37	151+21

Observers.	Notes.
6 observers	λ Taurid.
4 observers	Radiant not certain.
F.T.Naish, W.E.Yerward-James	λ Taurid.
J.M.Offord, W.A.Robson	Offord's observation by photograph.
Col G.L.Tupman, E.W.Barlow	ϵ Taurid.
10 observers	ϵ Taurid.
A.S.H., W.F.D.	ϵ Taurid.
A.S.H., W.F.D.	ι Aurigid.
3 observers	Brilliant θ Aurigid.
A.S.H., W.F.D.	ϵ Taurid.
C.L.B., J.P.Kenyon	Draconid.
A.S.H., W.F.D.	Leonid.
A.S.H., J.C.W.Herschel	Erratic Leonid.
J.C.W.Herschel, W.F.D.	ζ Hydrid.
Sir W.J.Herschel, W.E.B.	Leonid.
A.S.H., W. Wickham, and W.H.Robinson	β Leonid.
Rev. W.F.A.Ellison, G.Parkinson	Leonid.
Mrs. A. Brook, W.F.D.	Leonid.
C.L.B., Mrs. A. Brook, and others	Bright Leonid.
Sir W.J.Herschel, W.C.Tetley, and others	Bright Comæ Berenicid.
A.K., A.M.Davis	Bright Leonid.
T.H.A., Mrs. A. Brook	Erratic Leonid.
A.S.H., T.H.A., Mrs. A. Brook	Erratic Leonid.
A.S.H., T.H.A., P.M.Ryves	
A.S.H., T.H.A., W.H.Robinson	Leonid.
S.J.J., W.H.Robinson	Leonid.
W.H.Robinson, W.F.D.	Leonid.
J.H.B., C.L.B.	Leonid.
A.S.H., J.H.B.	Bright Leonid.
J.H.B., W.H.Robinson	Leonid.
T.W.B., A.Sullivan	Fine Taurid.
A.S.H., T.H.A.	
C.L.B., W.F.D.	Leonid.
Rev. W.F.A.Ellison, J.M'Harg	Fine Taurid.
A.S.H., P.M.Ryves	Leonid.

Date.	G.M.T.		Mag.	App. miles.	Dis. miles.	Path. miles.	Vel. m.p.s.	Radiant.	
	h	m						°	°
1903 Nov. 15	13	59	$2 \times \varphi - 3 \times \varphi$	90	53	64	...	149	+28
1903 Nov. 15	15	59½	4-2	91	45	128	29	147	-11
1903 Nov. 15	16	45	$\varphi - \varphi$	77	52	30	60	151	+25
1899 Nov. 15	16	49	> φ	83	50	43	...	150	+20
1899 Nov. 15	16	50	φ	82	45	49	...	151	+20
1903 Nov. 15	17	14½	φ	77	52	30	45	152	+25
1903 Nov. 15	17	23	$\varphi - \varphi$	84	63	27	67	149	+19
1903 Nov. 15	18	7	1- φ	81	60	24	44	151	+23
1907 Nov. 16	12	15½	= φ	85	56	83	swift	152	+23
1910 Nov. 16	12	26½	φ	91	45	165	30	150	+23
1910 Nov. 16	12	19	φ	95	58	40	40	26	+70
1908 Nov. 16	18	32	φ	85	50	41	...	150	+23
1908 Nov. 17	11	9¼	1	77	66	91	30	150	+21
1903 Nov. 18	12	44	$\varphi - \frac{1}{2}\varphi$	75	33	51	33	50	+20
1899 Nov. 19	8	5½	φ	85	42	68	23	60	+28
1910 Nov. 20	9	7	> φ	74	34	42	12	64	+25
1901 Nov. 23	10	27	$3 \times \varphi$	70	25	57	...	65	+23
1910 Nov. 25	6	55	1	68	47	51	14	65	+24
1910 Nov. 25	7	30	> 4	88	41	93	23	64	+21
1900 Nov. 27	11	10	> φ	57	17	40	...	47	+45
1901 Dec. 4	5	36	= φ	91	50	64	21	263	+36
1897 Dec. 9	9	47	> φ	76	21	90	...	113	+32
1903 Dec. 9	9	56½	> 1	91	58	54	...	110	+30
1907 Dec. 11	9	5	> φ	80	50	...	...	Geminid	
1903 Dec. 11	10	43	3-2	67	59	48	32	127	-2
1911 Dec. 11	7	29	> 1	74	45	82	15	108	+32
1902 Dec. 12	7	20	> 1- φ	63	42	31	14	43	+22
1897 Dec. 12	8	6	> φ	112	19	151	30	80	+23
1911 Dec. 12	5	56	> 1	54	43	94	14	92	+22
1900 Dec. 13	10	18½	> 1	59	44	24	..	109	+24
1901 Dec. 16	5	45	> 1-½ φ	68	54	51	20	105	+37
1904 Dec. 18	4	38	φ	81	30	64	20	337	+60
1910 Dec. 21	9	45	2	58	55	76	11	301	+24
1904 Dec. 22	4	30	> φ	83	28	58	29	330	+36
1911 Dec. 25	9	8	> 1	74	56	97	25	164	+33
1905 Dec. 30	4	27	½= φ	67	27	72	15	40	+23

Observers.

Notes.

C.L.B., W.F.A.Ellison, J.M'Harg.	Brilliant Leonid.
F.H.Wright, W.Parkinson, W.F.D.	Long-pathed Hydrid.
A.S.H., W.F.D.	Brilliant Leonid.
Rev. R. Killip, A.L. Cortie, and others	Brilliant Leonid.
3 observers	Brilliant Leonid.
A.S.H., W.F.D.	Brilliant Leonid.
A.S.H., H.C.	Brilliant Leonid.
A.S.H., A.K., W.F.D.	Brilliant Leonid.
W.H. Milligan, H. Nevison	Brilliant Leonid.
J.M'Harg, Miss Metcalfe, and others	Brilliant Leonid. Streak 3 mins.
E.C.Arthy and another	
E.Hawks and another	Brilliant Leonid.
C.L.B., J.P.Kenyon	Leonid. Very slow velocity.
A.S.H., C.L.B.	Taurid.
C.L.B., T.W.B.	Taurid.
3 observers	Taurid.
3 observers	Taurid.
W.F.D., J. Hicks	Taurid.
J. Hicks, W.A. Robson	Taurid.
A.S.H. and 2 others	Brilliant $\alpha - \beta$ Perseid.
62 observers	Splendid fireball.
E.H.Cullum and others	A fine Geminid.
C.L.B., J.P.Kenyon	Bright Geminid.
3 observers	Geminid fireball.
C.L.B., H.C.	Monocerotid.
Mr. and Mrs. W., Miss H.M. Metcalfe	Bright Geminid.
T.H.A., S.J.J.	ϵ Arietid.
E.H.Cullum and others	ζ Taurid.
Mr. and Mrs. W., W.F.D.	η Geminid.
T.H.A., A.K.	δ Geminid.
W.E.B., W.A.H., and others	Geminid fireball.
F.L.Raymond, H.M. Whitley	Cepheid.
W.F.D., F.T.Naish	Very late Sagittid.
6 observers	π Pegasid.
Mr. and Mrs. W., E.H. Allsop	Bright ξ Ursid.
25 observers	ϵ Taurid.

ADDENDA.

Date.	G.M.T.	Mag.	App.	Dis.	Path.	Vel.	Radiant.	
	h m		miles.	miles.	miles.	m.p.s.	°	°
1901 Apr. 19	9 28	$\eta - > \varphi$	56	52	40	25	226	- 17
1901 Apr. 20	10 20	2 - 1	58	52	26	16	226	- 17
1897 Apr. 22	10 25	2 - η	85	30	58	9	161	+ 59
1901 May 16	9 52	1	67	50	78	26	350	+ 48
1910 May 2	14 40	η	67	46	137	34	330	0
1899 Nov. 13	15 13½	3 - 2½	65	55	40	57	130	- 0
1904 Nov. 15	12 28	$\eta - \varphi$	89	69	66	57	151	+ 23
1904 Nov. 15	14 40	4 × φ	86	40	88	...	31	+ 20
1902 Nov. 23	10 20	2	67	40	35	...	57	+ 11

ADDENDA.

Observers.	Notes.
J. H. B. and others	
C. L. B., J. C. W. Herschel	
D. B., E. R. Blakeley	Radiant uncertain.
G. T. Davis, G. H. Price	
E. Rowsell, etc.	Aquarid.
A. S. H., J. H. B.	Southern Monocerotid.
C. L. B., A. K.	Bright Leonid.
T. W. B., Mrs. A. Brook	Bright α Arietid and others.
Mrs. A. B., C. L. B.	

Note.—1900 May 3 (p. 431). Well-observed Aquarid from (?) Halley's Comet. The motion observed, much slower than the theoretical velocity of 42 miles per second.

